Introduction to Brushless Motor Parameters and Figures of Merit

Motor parameters can be confusing to understand and properly apply. The cause of this is that different vendors apply definitions in different ways, incorrectly at times. Without understanding motor types, units of measure, windings and drive calculations, figures of merit will be wrong and invariably lead to bad decisions in the design of servo systems. It is my intent to clear these matters up in an unambiguous manner.

Document Organization:

- **In Introduction to Brushless Motor Parameters and Figures of Merit** (Main Document)
    - Appendix 1 Squarewave calculation of the Fourier Series peak and total rms value.
    - Appendix 2 Trapazoidal calculation of the Fourier Series peak and total rms value.
- **Attachment A**: Torque Calculations, Parameter Comparisons and Chart Summaries 1 and 2.
- **Example 1**: Delta Motor -- Sine / Sine and Sine / Square Evaluation
- **Example 2**: Wye Motor -- Sine / Sine Evaluation
- **Example 3**: Wye Motor -- Square / Square and Square / Sine Evaluation

The Examples are provided to verify equations and claims.
In the Examples when referencing equation numbers from the main document, (#) is used and from Attachment A, (A#) is used. For example;

(A5) would reference Attachment A, Equation (5).

Motor Types:
Before we begin it is important to discuss brushless motor types. In spite of literature to the contrary, there are basically two types of brushless motors. AC Brushless (sinusoidal back-emf) and DC Brushless (trapezoidal back-emf). It is the motor commutation drive which determines how each type will perform.

The vast majority of brushless motors manufactured today are 3-phase (3Ø), non-salient permanent magnet synchronous motors. (Saliency is a feature whereby winding inductance varies a function of angle and the torque is not proportional to current. We will not deal with salient motors).

Units of Measure:
Unless otherwise indicated parameters and figures of merit are in SI units (The International System of Units i.e. metric units). Conversions from English units to SI units *before* calculations or from SI units to English units *after* calculations is assumed. All parameter relationships presented are only valid when SI units are employed. Nomenclature – I use subscript ϕ to indicate a phase quantity and subscript ϕϕ for line to line quantities.

Windings:
Only 3-phase windings are discussed herein. There are two winding topologies –
WYE and DELTA. All equations presented herein are usable for both types of windings.
This will be verified by using these equations with both types of windings in the attached Examples.

Symbols:

Symbol	**Interpretation**	**Units**
V	Voltage (Potential difference)	Volts (V)
I	Current	Amps (A)
R	Resistance	ohms (Ω)
L	Inductance	Henries (Wb / A(turn)
pf	Power factor	
P_X	Power dissipated by X	Watts (W)
τ	Torque	Newton Meter (Nm)
τ_m, τ_e	Time constants	Seconds (s)
t	Time	Seconds (s)
ω	Angular velocity	(rad/s)
α	Angular acceleration	(rad/s/s)
ϴ	Angle mechanical	(radians or degrees)
P	Number of magnetic poles	(P/2 = Number of pole pairs)
e_{Bxx}	Back emf per xx (phase or phase to phase)	(Volts / rad/s)
J	Moment of Inertia	(kg m^2)
K_{Tx}	torque constant per subscript	(Nm / A_{peak})
K_{eX}	velocity constant per subscript	(V_{peak} / rad/s)
K_T	Total torque constant	various dimensions
X	general quantity or don't care	

Subscripts X

accl	-	acceleration
p	-	peak
rms	-	root mean square
ϕ	-	phase value
ϕϕ	-	phase to phase (line-line) value
R, LT, T	-	resistor power, load torque power and total power dissipation.
S	-	Motor Drive = Sinewave
Q	-	Motor Drive = Squarewave
SS	-	Motor / Drive = Sine / Sine
QQ	-	Motor / Drive = Square / Square
SQ	-	Motor / Drive = Sine / Square
QS	-	Motor / Drive = Square / Sine

Drive Commutation Control Methods:

The most common commutation control methods are often identified as 1. Sinusoidal (Sinewave commutation) and 2. 6-step or Block commutation (Squarewave commutation) where each electrical cycle (one electrical cycle/permanent magnet pole-pair) is mapped into six commutation steps.

ω_{mech} = mechanical rotation speed

ω_{elect} = electrical radian frequency

P = number of magnetic poles

$\frac{P}{2}$ = number of magnetic pole pairs

Then relating electrical commutation speed to mechanical rotation speed is given by:

$$\omega_{elect} = \omega_{mech} \cdot \frac{P}{2} \qquad (1)$$

Note: Field Oriented Control (FOC) is viewed as a special case of sinusoidal commutation.

Sinewave Drive

In a sinewave drive both the current and back emf (Bemf) are ideally sinusoidal. Continuous position feedback (i.e. optical encoder) is usually employed but sometimes sensorless techniques are used. Power is delivered by a 6-transistor (now mostly MOSFET) switching system connected as 3 half bridge stages (top and bottom switches connected to each motor lead) Refer to Figure 1.

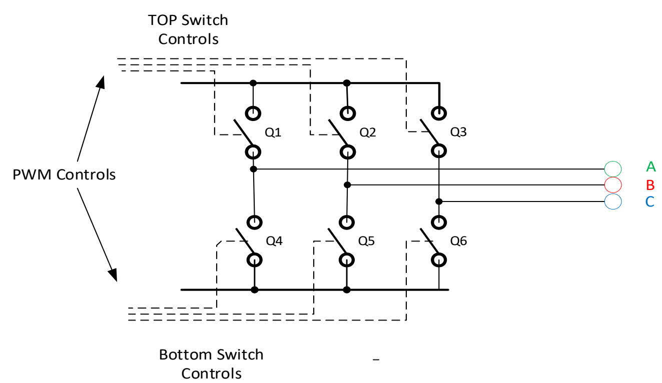
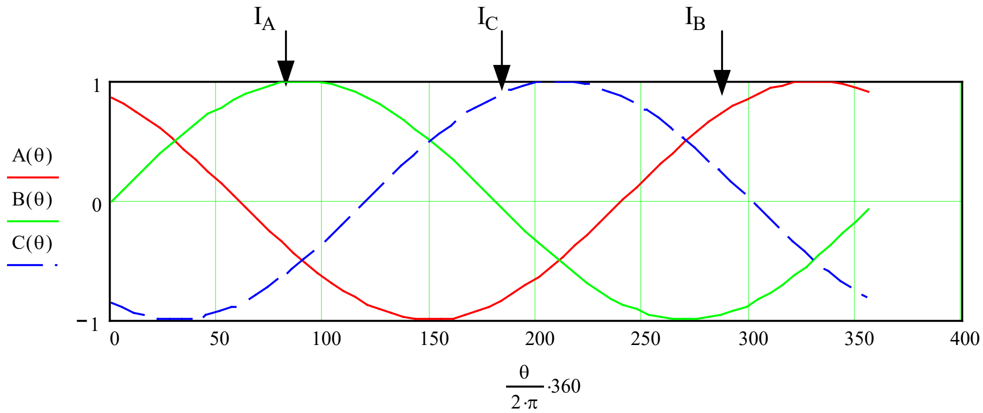


Figure 1
Simplified 3 Phase Drive Electronics

At any given time *one member of each half bridge is conducting, i.e.* ***all windings are used****.* However, each motor lead's current is shifted from the other by 120 electrical degrees. PWM is employed which is modulated to provide the sinusoidal waveshape. Sinusoidal waveshapes are shown in Figure 2.



Electrical degrees

Figure 2.

Sinusoidal Current Drive

Where τ represents torque.

$$\tau_{Total} = \frac{3}{2}\cdot\tau_{\phi} \qquad (2)$$

Where τ_{ϕ} is the phase torque.

The total torque, $t_{Total,}$ Is ideally constant.

Squarewave Drive

This drive's current is squarewave with a dead zone crossover point. See Figure. 3. For best results the Bemf should be trapezoidal in shape. The use of three half-bridge switches is used. but **only two** windings are active at a given time (exception is the Delta equivalence discussed later). PWM is employed in a chopper mode. Hall Effect sensors are typically employed for position feedback which is used to control the switch commutation sequence.

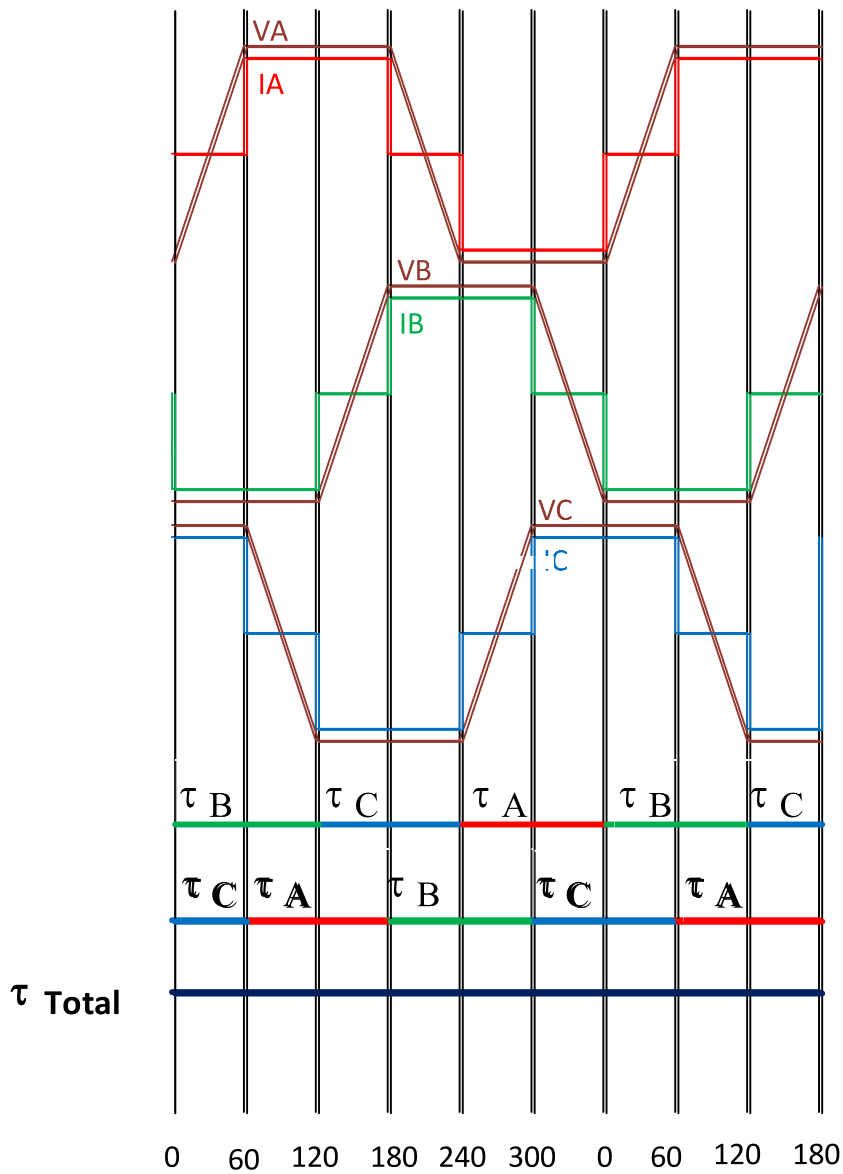


Figure 3.
Squarewave Commutation

$$\tau_A = \tau_B = \tau_C = \tau_\phi \qquad (3)$$

$$\tau_{Total} = \sum_{i=1}^{6} \left(\tau_{Ai} + \tau_{Bi} + \tau_{Ci}\right) = 2 \cdot \tau_\phi \qquad (4)$$

Also, $t_{Total,}$ is ideally constant.

Analysis

Note:

Whether rms or peak --
The phase voltage is always given by:

$$V_\phi = e_{B\phi} + I_\phi \cdot R_\phi + L_\phi \cdot \frac{d}{dt} I_\phi \qquad (5)$$

or

$$V_\phi = K_{e\phi} \cdot \omega + I_\phi \cdot R_\phi + L_\phi \cdot \frac{d}{dt} I_\phi \qquad (6)$$

where

$$K_{e\phi} = \frac{e_{B\phi}}{\omega} \qquad (7)$$

Likewise, for the phase-phase voltage

$$V_{\phi\phi} = e_{B\phi\phi} + I_{\phi\phi} \cdot R_{\phi\phi} + L_{\phi\phi} \cdot \frac{d}{dt} I_{\phi\phi} \qquad (8)$$

or

$$V_{\phi\phi} = K_{e\phi\phi} \cdot \omega + I_{\phi\phi} \cdot R_{\phi\phi} + L_{\phi\phi} \cdot \frac{d}{dt} I_{\phi\phi} \qquad (9)$$

where

$$K_{e\phi\phi} = \frac{e_{B\phi\phi}}{\omega} \qquad (10)$$

First, we define the Motor Parameters and then the Figures of Merit. A summary of Example results is provided.

Motor Parameters:

$R_{\phi\phi Y} = 2\ R_{\phi Y}$	Line to line resistance of a WYE winding 3 phase motor as a function of the phase resistance.
$R_{\phi\phi\Delta} = 2/3\ R_{\phi\Delta}$	Line to line resistance of a DELTA winding 3 phase motor as a function of the phase resistance.
$L_{\phi\phi Y} = 2\ L_{\phi Y}$	Line to line inductance of a WYE wound 3 phase motor as a function of the phase inductance.
$L_{\phi\phi\Delta} = 2/3\ L_{\phi\Delta}$	Line to line inductance of a DELTA wound 3 phase motor as a function of the phase inductance.
$Ke_{\phi\phi p} = eB_{\phi\phi p} / \omega$	Back emf line to line constant calculated from line to line peak voltage.
$Ke_{\phi p} = eB_{\phi p} / \omega$	Back emf phase constant calculated from the phase peak voltage. *Non- measurable quantity.*
$K_{Tp} = \tau / I_{\phi\phi p}$	Overall generic torque constant as a function of peak phase to phase current.
$K_{Trms} = \tau / I_{\phi\phi rms}$	Overall generic torque constant as a function of rms phase to phase current. Waveform dependent.
$Ke_{\phi\phi rms} = eB_{\phi\phi rms} / \omega$	Generic back emf line to line constant calculated from line to line rms voltage. Waveform dependent.
$Ke_{\phi rms} = eB_{\phi rms} / \omega$	Generic back emf phase constant calculated from the phase rms voltage. Waveform dependent. *Non- measurable quantity.*
J or J_M	Motor moment of inertia (ignores load or gearbox and load). Impacts the angular acceleration.

$$\tau_{accl} = J \cdot \alpha = J \cdot \frac{d}{dt}\omega \qquad (11)$$

τ_{accl}	is the acceleration torque	$N \cdot m$
α	is the motor shaft acceleation	$\frac{rad}{s^2}$

Figures of Merit:

1. Torque / Inertia ratio – τ / J.
2. Resistive power loss (drive dependent). $P_{R.}$
3. Power rate PR = τ^2 / J.
4. Stall torque – $\tau_{stall.}$ (drive dependent).
5. No load Speed – ω_{NL}
6. Steepness = Stall torque / No load speed $\tau_{stall.} / \omega_{NL.}$ (drive dependent) = 1 / R_m
   R_m is the speed vs torque curve slope.
7. Motor Constant – $K_M = \tau$ / SQRT(P_R). (drive dependent).
   SQRT(x) stands for square root of (x)
8. Motor mechanical time constant (drive dependent) - $\tau_{m.}$
9. Speed Rate or reciprocal of motor's mechanical time constant (drive dependent) – SR = 1 / $\tau_{m.}$
10. Electrical time constant - $\tau_e = L_{\phi\phi} / R_{\phi\phi}$.

Assumptions:

- Temperature effects are ignored.
- Winding inductance is ignored in calculations.
- Power conversions are assumed to be lossless (ideal).
- No friction or damping considered.
- All 3 Phase windings are balanced. Which means all impedances are identical (same power factor) and voltages and currents are equal in magnitude but differ in identical phase shifts (Phase voltage to phase voltage and phase current to phase current).

Summary, Approach, and Examples:

Attachment A provides a summary of equations and calculations. Though to this point only Motor / Drive combinations of sinewave / sinewave (SS) and squarewave / squarewave (QQ) have been mentioned, two (2) cross types have been added, which are sine / square (SQ) and square / sine (QS).

Note that for specific motor / drive configurations the SS, QQ, SQ and QS subscripts are used.
For a motor drive alone S and Q subscripts are used.

Approach:

To obtain the parameter constants and figures of merit, the approach is to identify the load power and resistive power loss for each configuration. From these the torque and back emf effects can be quantified.

For all 3 phase systems the total power P_T is equal to the sum of the load power, P_{LT} and resistor power loss, P_R. Note power factor = 1.

$$P_T = P_{LT} + P_R \qquad (12)$$

Sinewave drive (Δ or Y) –

$$P_{TS} = 3 \cdot e_{B\phi rms} \cdot I_{\phi rms} + 3 \cdot R_\phi \cdot I_{\phi rms}^2 \qquad (13)$$

For a **WYE (Y)**

$$\frac{e_{B\phi\phi}}{\sqrt{3}} = e_{B\phi} \qquad I_{\phi\phi} = I_\phi$$

$$\frac{R_{\phi\phi}}{2} = R_\phi$$

For a **Delta (Δ)**

$$e_{B\phi\phi} = e_{B\phi} \qquad \frac{I_{\phi\phi}}{\sqrt{3}} = I_\phi$$

$$\frac{3}{2} \cdot R_{\phi\phi} = R_\phi$$

Substituting the phase quantities with phase to phase values yields for either **Y or Δ,**
Load power:

$$P_{LTS} = \omega \cdot \tau = \sqrt{3} \cdot e_{B\phi\phi rms} \cdot I_{\phi\phi rms} = \frac{\sqrt{3}}{2} \cdot e_{B\phi\phi p} \cdot I_{\phi\phi p} \qquad (14)$$

Resistor power loss:

$$P_{RS} = \frac{3}{2} \cdot R_{\phi\phi} \cdot I_{\phi\phi rms}^2 \qquad (15)$$

Squarewave drive –

For a **WYE (Y)**, Only two phases active at a time. Voltage and current are DC (peak) values for each 60 -degree interval.

$$P_{TQ} = 2 \cdot e_{B\phi p} \cdot I_{\phi p} + 2R_{\phi} \cdot I_{\phi p}{}^{2} \qquad (16)$$

$$e_{B\phi\phi} = 2 \cdot e_{B\phi} \qquad I_{\phi\phi} = I_{\phi}$$

$$\frac{R_{\phi\phi}}{2} = R_{\phi}$$

Substituting the Y relationships yields,

$$P_{LTQ} = e_{B\phi\phi p} \cdot I_{\phi\phi p} \qquad (17)$$

$$P_{RQ} = R_{\phi\phi} \cdot I_{\phi\phi p}{}^{2} \qquad (18)$$

For **a Delta (Δ)** all three phases carry current.

$$P_{LTQ} = e_{B\phi ap} \cdot I_{\phi ap} + \left(e_{B\phi bp} + e_{B\phi cp}\right) \cdot \left(I_{\phi ap} - I_{\phi\phi p}\right) \qquad (19)$$

Phase to phase relationships for the Delta, using the a, b and c to represent the individual winding legs.

$$e_{B\phi ap} = -\left(e_{B\phi bp} + e_{B\phi cp}\right)$$

$$I_{\phi ap} = \frac{2 \cdot I_{\phi\phi p}}{3} \qquad e_{B\phi\phi p} = e_{B\phi p}$$

$$I_{\phi bp} = I_{\phi cp} = \frac{-I_{\phi\phi p}}{3} = \left(I_{\phi ap} - I_{\phi\phi p}\right)$$

Putting these together yields

$$P_{LTQ} = e_{B\phi\phi p} \cdot I_{\phi\phi p} \qquad (20)$$

$$P_{RQ} = R_{\phi\phi} \cdot I_{\phi\phi p}{}^{2} \qquad (21)$$

Which are identical to equations 17 and 18 in form.

Therefore, we use the following with squarewave drives for **WYE or Delta windings**,

$$P_{TQ} = e_{B\phi\phi p}\cdot I_{\phi\phi p} + R_{\phi\phi}\cdot I_{\phi\phi p}{}^{2} \qquad (22)$$

$$P_{LTQ} = \tau\cdot\omega = e_{B\phi\phi p}\cdot I_{\phi\phi p} \qquad (23)$$

$$P_{RQ} = R_{\phi\phi}\cdot I_{\phi\phi p}{}^{2} \qquad (24)$$

For the following Power Section please refer to Appendix 1 and Appendix 2 for Peak to RMS conversions for Current and Voltage.

Motor / Drive = Sine / Sine from (14) and (15).

$$P_{LTSS} = \tau_{SS}\cdot\omega = \frac{\sqrt{3}}{2}\cdot e_{B\phi\phi p}\cdot I_{\phi\phi p} = \frac{\sqrt{3}}{2}\cdot K_{e\phi\phi p}\cdot\omega\cdot I_{\phi\phi p} \qquad (25a)$$

$$P_{LTSS} = \sqrt{3}\cdot K_{e\phi\phi rms}\cdot\omega\cdot I_{\phi\phi rms} \qquad (25b)$$

where $K_{e\phi\phi p}$ is defined as $K_{e\phi\phi p} = \dfrac{e_{B\phi\phi p}}{\omega}$

Dividing by ω gives,

$$\frac{P_{LTSS}}{\omega} = \tau_{SS} = \frac{\sqrt{3}}{2}\cdot K_{e\phi\phi p}\cdot I_{\phi\phi p} \qquad \textbf{(26)}$$

$$P_{RSS} = \frac{3}{2}\cdot R_{\phi\phi}\cdot I_{\phi\phi rms}{}^{2} \qquad \textbf{(27)}$$

Motor / Drive = Square / Square, from (23) and (24).

$$P_{LTQQ} = \tau_{QQ}\cdot\omega = e_{B\phi\phi p}\cdot I_{\phi\phi p} = K_{e\phi\phi p}\cdot\omega\cdot I_{\phi\phi p} \qquad (28a\ )$$

$$P_{LLQQ} = \sqrt{3}\cdot\sqrt{\frac{9}{10}}\cdot K_{e\phi\phi rms}\cdot\omega\cdot I_{\phi\phi rms} \qquad (28b)$$

$$\frac{P_{LTQQ}}{\omega} = \tau_{QQ} = K_{e\phi\phi p}\cdot I_{\phi\phi p} \qquad \textbf{(29)}$$

$$P_{RQQ} = R_{\phi\phi}\cdot I_{\phi\phi p}{}^{2} \qquad \textbf{(30)}$$

Motor / Drive = Sine / Square use (14) and (24) with modification**.**

Only the fundamental component of drive current contributes to the torque and therefore load power.

SEE Appendix 1 for derivation of F_I.

$$P_{LTSQ} = \tau_{SQ} \cdot \omega = \frac{\sqrt{3}}{2} \cdot e_{B\phi\phi p} \cdot F_I \cdot I_{\phi\phi p} \qquad (31)$$

F_I – is the Fourier Series modifier to obtain the peak current of the fundamental sinusoidal component.

$$I_{\phi\phi p}{}^{sine} = F_I \cdot I_{\phi\phi p} \qquad (56) \qquad F_I = \frac{2 \cdot \sqrt{3}}{\pi} \qquad (57)$$

$$P_{LTSQ} = \frac{3}{\pi} \cdot e_{B\phi\phi p} \cdot I_{\phi\phi p} = \frac{3}{\pi} \cdot K_{e\phi\phi p} \cdot \omega \cdot I_{\phi\phi p} \qquad (32a)$$

$$P_{LTSQ} = \frac{3}{\pi} \cdot \sqrt{3} \cdot K_{e\phi\phi rms} \cdot \omega \cdot I_{\phi\phi rms} \qquad (32b)$$

$$\frac{P_{LTSQ}}{\omega} = \tau_{SQ} = \frac{3}{\pi} \cdot K_{e\phi\phi p} \cdot I_{\phi\phi p} \qquad (33)$$

$$P_{RSQ} = R_{\phi\phi} \cdot I_{\phi\phi p}{}^2 \qquad (34)$$

Motor / Drive = Square / Sine use (14) and (15) with modification.

Here only the fundamental component of the back emf contributes to the torque and load power.

SEE Appendix 2 for the derivation of F_V.

$$P_{LTQS} = \tau_{QS} \cdot \omega = \frac{\sqrt{3}}{2} \cdot F_V \cdot e_{B\phi\phi p} \cdot I_{\phi\phi p} \qquad (35)$$

F_V – is the Fourier Series modifier to obtain the peak back emf voltage of the fundamental sinusoidal component.

$$e_{B\phi\phi p}{}^{sine} = F_V \cdot e_{B\phi\phi p} \qquad (62) \qquad F_V = \frac{6 \cdot \sqrt{3}}{\pi^2} \qquad (63)$$

$$P_{LTQS} = \left(\frac{3}{\pi}\right)^2 \cdot e_{B\phi\phi p} \cdot I_{\phi\phi p} = \left(\frac{3}{\pi}\right)^2 \cdot K_{e\phi\phi p} \cdot \omega \cdot I_{\phi\phi p} \qquad (36a)$$

$$P_{LQS} = \sqrt{3} \cdot K_{e\phi\phi rms} \cdot \omega \cdot I_{\phi\phi rms} \qquad (36b)$$

$$\frac{P_{LTQS}}{\omega} = \tau_{SS} = \left(\frac{3}{\pi}\right)^2 \cdot K_{e\phi\phi p} \cdot I_{\phi\phi p} \qquad (37)$$

$$P_{RQS} = \frac{3}{2} \cdot R_{\phi\phi} \cdot I_{\phi\phi rms}{}^2 \qquad (38)$$

Now we have all the ingredients to derive parameter constants and calculate figures of merit. What one cannot overlook is the commutation drive (winding) conversions.

Figure of Merit Calculations:

We will now look at τ_{stall}, ω_{NL}, R_{m}, K_{M}, τ_{m} and τ_{e} formulas.

Stall Torque -

$$e_{B\phi} = e_{B\phi\phi} = 0 \qquad (39)$$

Let $V_{\phi\phi ptest}$ be the terminal voltage.

with $$\tau_{stall} = K_{Tp} \cdot I_{\phi\phi pstall} \qquad (39a)$$

or $$\tau_{stall} = K_{Trms} \cdot I_{\phi\phi rmsstall} \qquad (39b)$$

For **sinewave drives**,

Y winding

$$I_{\phi pstall} = I_{\phi\phi pstall} = \frac{V_{\phi ptest}}{R_{\phi}} = \frac{\frac{V_{\phi\phi ptest}}{\sqrt{3}}}{\frac{R_{\phi\phi}}{2}}$$

$$I_{\phi\phi pstall} = \frac{2}{\sqrt{3}} \frac{V_{\phi\phi ptest}}{R_{\phi\phi}}$$

Δ winding

$$I_{\phi\phi pstall} = \sqrt{3} \cdot I_{\phi pstall} = \sqrt{3} \frac{V_{\phi ptest}}{R_{\phi}} = \sqrt{3} \cdot \left(\frac{V_{\phi\phi ptest}}{\frac{3}{2} \cdot R_{\phi\phi}} \right) \qquad V_{\phi} = V_{\phi\phi}$$

$$I_{\phi\phi pstall} = \frac{2}{\sqrt{3}} \frac{V_{\phi\phi ptest}}{R_{\phi\phi}}$$

Therefore, for all **sinewave drives (Y or Δ)**

$$I_{\phi\phi pstallS} = \frac{2}{\sqrt{3}} \cdot \frac{V_{\phi\phi ptest}}{R_{\phi\phi}} \qquad (40) \qquad \tau_{stallS} = \frac{2}{\sqrt{3}} \cdot \frac{V_{\phi\phi ptest}}{R_{\phi\phi}} \cdot K_{TpS} \qquad (100)$$

For **squarewave drive**s,

Y winding

$$I_{\phi pstall} = I_{\phi\phi pstall} = \frac{V_{\phi ptest}}{R_{\phi}} = \frac{\frac{V_{\phi\phi ptset}}{2}}{\frac{R_{\phi\phi}}{2}}$$

$$I_{\phi\phi pstall} = \frac{V_{\phi\phi ptest}}{R_{\phi\phi}}$$

Δ winding - Refer to page 10, phase to phase relationship for the Delta.

$$I_{\phi\phi pstall} = \frac{3}{2}\cdot I_{\phi pstall} = \frac{3}{2}\cdot\left(\frac{V_{\phi ptest}}{R_{\phi}}\right) = \frac{3}{2}\cdot\left(\frac{V_{\phi\phi ptest}}{\frac{3}{2}\cdot R_{\phi\phi}}\right) \qquad V_{\phi} = V_{\phi\phi}$$

$$I_{\phi\phi pstall} = \frac{V_{\phi\phi ptest}}{R_{\phi\phi}}$$

Therefore, for all **squarewave drives (Y or Δ)**

$$I_{\phi\phi pstallQ} = \frac{V_{\phi\phi ptest}}{R_{\phi\phi}} \qquad (41) \qquad \tau_{stallQ} = \frac{V_{\phi\phi ptest}}{R_{\phi\phi}}\cdot K_{TpQ} \qquad (101)$$

No load speed for any drive is given by,

$$\omega_{NL} = \frac{V_{\phi\phi ptest}}{K_{e\phi\phi p}} = \frac{V_{\phi\phi rmstest}}{K_{e\phi\phi rms}} \qquad (42)$$

Slope constant
Refer to Attachment A, Chart 2 Col 8 for the following;

$$R_m = \omega_{NL} / \tau_{stall} \qquad (43)$$

Solving for a **sinewave drive**,

$$R_{mS} = \frac{\sqrt{3}}{2}\cdot\frac{R_{\phi\phi}}{K_{e\phi\phi p}\cdot K_{TpS}} \qquad (44)$$

Solving for a **squarewave** drive,

$$R_{mQ} = \frac{R_{\phi\phi}}{K_{e\phi\phi p} \cdot K_{TpQ}} \qquad (45)$$

These two forms are often confused and the result is a 15.47% error.

***Speed Torque Curve** is given by,*

$$\omega(\tau) = \omega_{NL} - R_m \cdot \tau \qquad (46)$$

The **Motor Constant**, defined as,

$$K_M = \frac{\tau}{\sqrt{P_R}} \qquad (47) \qquad \text{Rearranging} \qquad P_R = \left(\frac{\tau}{K_M}\right)^2 \qquad (102)$$

with $\tau = K_{Trms} \cdot I_{\phi\phi rms}$ and $\tau = K_{Tp} \cdot I_{\phi\phi p}$

For a **sinewave drive**, Refer to equations, (27) and (38)

$$\sqrt{P_{.R}} = \sqrt{\frac{3}{2}} \sqrt{R_{\phi\phi}} \cdot I_{\phi\phi rms}$$

Solving for $K_{M,}$

$$K_{MS} = \sqrt{\frac{2}{3}} \cdot \frac{K_{TrmsS}}{\sqrt{R_{\phi\phi}}} \qquad (48)$$

For a **squarewave drive**, Refer to equations, (30) and (34)

$$\sqrt{P_R} = \sqrt{R_{\phi\phi}} \cdot I_{\phi\phi p}$$

Solving for K_M,

$$K_{MQ} = \frac{K_{TpQ}}{\sqrt{R_{\phi\phi}}} \qquad (49)$$

Mechanical time constant τ_m for a Y winding is defined as,

Y winding

$$\tau_m = \frac{R_\phi \cdot J}{K_{e\phi p} \cdot K_{Tp}} \qquad (50)$$

Performing the ϕ to ϕϕ conversions for a Y winding, sinewave drive yields,

$$K_{e\phi p} = \frac{K_{e\phi\phi p}}{\sqrt{3}} \qquad R_\phi = \frac{R_{\phi\phi}}{2}$$

$$\tau'_m = \frac{\sqrt{3}}{2} \cdot \frac{R_{\phi\phi} \cdot J}{K_{e\phi\phi p} \cdot K_{Tp}}$$

Mechanical time constant τ_m for a Δ winding is given by,

Δ winding

$$\tau_m = \frac{R_\phi \cdot J}{K_{e\phi\phi p} \cdot K_{Tp\phi}} \qquad (51)$$

$$K_{TP\phi} = \sqrt{3} K_{Tp} \qquad R_\phi = \frac{3R_{\phi\phi}}{2}$$

Therefore, for a **sinewave drive with a Δ winding** and using (51),

$$\tau'_m = \frac{\sqrt{3}}{2} \cdot \frac{R_{\phi\phi} \cdot J}{K_{e\phi\phi p} \cdot K_{Tp}}$$

Therefore, for a **sinewave drive**, since Y and Δ are identical,

$$\tau_{mS} = \frac{\sqrt{3}}{2} \cdot \frac{R_{\phi\phi} \cdot J}{K_{e\phi\phi p} \cdot K_{TpS}} \qquad (52)$$

Proceeding in like fashion for a squarewave drive using (50),

Y winding

$$K_{e\phi p} = \frac{K_{e\phi\phi p}}{2} \qquad R_\phi = \frac{R_{\phi\phi}}{2}$$

$$\tau_m = \frac{R_{\phi\phi} \cdot J}{K_{e\phi\phi p} \cdot K_{Tp}}$$

Squarewave drive using (51),
Refer to page 10 for current division.

Δ winding

$$K_{Tp\phi} = \frac{2}{3}\cdot K_{Tp} \qquad R_{\phi} = \frac{3R_{\phi\phi}}{2}$$

$$\tau_{m} = \frac{R_{\phi\phi}\cdot J}{K_{e\phi\phi p}\cdot K_{Tp}}$$

Therefore, for a **squarewave drive**, since <u>Y and Δ are identical</u>,

$$\tau_{mQ} = \frac{R_{\phi\phi}\cdot J}{K_{e\phi\phi p}\cdot K_{TpQ}} \qquad (53)$$

Note that the **electrical time constant** is found from,

$$\tau_{e} = \frac{L_{\phi\phi}}{R_{\phi\phi}} \qquad (54)$$

There are four (4) sections to **Attachment A** which are:

- Torque Calculations and Voltage and Current Conversions (1 page).
- Comparison of Parameters between Motor Configurations (2 pages).
- Chart 1, Parameters (generic) (1 page)
- Chart 2, Figures of Merit (generic) (1 page)

Chart parameter manipulation:

Note that manipulation of the columns can be used to obtain other ratios or reciprocals.
For example, using Chart 1, if one desires $K_{Trms} / K_{e\phi\phi p}$ multiply Col 1 and Col 2 together.

For a **sine /sine** Col 1 ($K_{Tp} / K_{e\phi\phi p}$) multiplied by Col 2 (K_{Trms} / K_{Tp}) yields –

$$\frac{K_{Tp}}{K_{e\phi\phi p}} \cdot \frac{K_{Trms}}{K_{Tp}} = \frac{K_{Trms}}{K_{e\phi\phi p}}$$

F*or Sine / Sine*

$$\frac{K_{Trms}}{K_{e\phi\phi p}} = \frac{\sqrt{3}}{2} \cdot \sqrt{2} = \sqrt{\frac{3}{2}}$$

If the inverse of a parameter relationship is sought; for example, **Sine / Sine** ratio of $K_{Trms} / K_{e\phi\phi rms}$ is found using the inverse of Col 3,

$$\left(\frac{K_{e\phi\phi rms}}{K_{Trms}}\right)^{-1} = \frac{K_{Trms}}{K_{e\phi\phi rms}} = \frac{1}{\frac{1}{\sqrt{3}}} = \sqrt{3}$$

Discussion:

The purpose of the Examples section is twofold --

1. Verify equations for both delta and wye windings for the following Motor / Drive configurations:
   Sinewave / Sinewave
   Squarewave / Squarewave
   Sinewave / Squarewave
   Squarewave / Sinewave
2. Verify the benefit of using a sinewave drive for either motor type; when operating with the same torque and at the same speed. At least from the perspective of resistor power loss.

All motors were selected at random only subject to the desired motor / drive configuration.

Methodology used –

1. Provide Conversion Factors for English to SI and SI to English units.
2. List datasheet specifications for torque constant, back emf constant, motor constant, motor inertia as well as mechanical and electrical time constants (if provided).
3. Check K_T and K_e conversion per Attachment A, Chart 1.
4. Find stall torque and no-load speed. Compare to published values.
5. Evaluate speed / torque curve for load power.
6. Calculate and check specs for Motor Constant (K_M) and time constants τ_m and τ_e.
7. Calculate resistor power and check load power expressions.

Example 1:

The point of Example 1 is to verify the equations for a Motor / Drive = Sine / Sine with a Delta winding. With the exception of τ_m calculation there was excellent agreement throughout. I attribute the τ_m discrepancy to a specification typo.

A benefit to the selection of the BEI DIP24-11-001A is the fact that squarewave drive parameters are provided. However, there is some issue here. K_{Tp} is listed as 62 oz-in /A (0.438 Nm / A). The case of a Motor / Drive = Sine / Square does NOT mean $K_{Tp} = K_{e\phi\phi p}$ (which is true only for a Square / Square). With the appropriate parameter adjustment ($K_{e\phi\phi p}$ = 0.438 Vp / rad/s) all equations were verified.
$K_{e\phi\phi p}$ is independent of motor drive. It is a motor type parameter.

Of note is the slight increase in τ_m by the factor of 3 / π. This is due to the same amount of decrease in K_{Tp}. See page 6 of Example 1. Also note that K_M is affected (reduced) because the **resistor power loss changes** (is increased).

Example 2:

Example 2's motor was selected to verify the WYE winding equations for a Sine / Sine Drive. A BEI DIP30-07-001A was chosen. A specification $K_{Tp} = K_{e\phi\phi p}$ = 0.226 V / rad/s, raised a flag. Using the Motor Constant (assuming it was correct) to solve for K_{Trms} got things on track. Given the corrections the mechanical time constants published and calculated matched! The only discrepancy was in the calculation of stall torque current. The stall torque value is correct, the current to achieve it is incorrect. The Vendor incorrectly used the squarewave relationship (41) instead of the sinewave (40) relationship. See Example 2, page 3.

Example 3:

The motor selected for Example 3 was a Kollomorgen TBM-6013-A. This selection would provide a challenge to the Motor / Drive = Square / Square and Square / Sine equations.

Square / Sine Analysis –

The only issue found was the Vendor's assumption that the parameter $K_{e\phi\phi rms} = K_{e\phi\phi p}$ / SQRT(2). This is not true in this case.

To elaborate, a trapezoidal Bemf waveform for a 6-step commutation would exhibit a peak sinusoidal fundamental of (Refer to Appendix 2), F_V = -- (63).

With

F_V – is the Fourier Series modifier to obtain the peak back emf voltage of the fundamental sinusoidal component.

$$e_{B\phi\phi p}{}^{sine} = F_V \cdot e_{B\phi\phi p} \qquad (62) \qquad F_V = \frac{6\cdot\sqrt{3}}{\pi^2} \qquad (63)$$

The rms of this sinusoid would be:

$$e_{B\phi\phi rms}{}^{sine} = \frac{F_V \cdot e_{B\phi\phi p}}{\sqrt{2}} = 0.74456 \cdot e_{B\phi\phi p}$$

The rms value of a periodic function, of period T, is found from,

$$f_{rms} = \sqrt{\frac{1}{T}\cdot\int_0^T f(t)^2\,dt}$$

For a trapezoid, with a peak value of $e_{B\phi\phi p}$ and 120-degree commutation,
This integration yields, (Appendix 2).

$$e_{B\phi\phi rms}{}^{trap} = \frac{\sqrt{5}}{3}\cdot e_{B\phi\phi p} \qquad (65)$$

Note that

$$\frac{e_{B\phi\phi rms}{}^{sine}}{e_{B\phi\phi rms}{}^{trap}} = 0.99892616$$

This suggests that most of the energy of the trapezoid is contained in the fundamental frequency.
The $e_{B\phi\phi p}$ divided by SQRT(2) = 0.70711 $e_{B\phi\phi p}$ = $e_{B\phi\phi rms}$ is therefore incorrect. Close (low by 5.41 %) but not accurate.
No mechanical (τ_m) or electrical (τ_e) time constant is listed for this motor.

Motor / Drive = Square / Sine –

The $K_{e\phi\phi p}$ value (0.087 V / rad/s) was used with the Square / Sine equations.
This is due to the independence of $K_{e\phi\phi p}$ from drive type mentioned earlier.
Verification that the torque increase by 6*SQRT(3) / π^2 by using a sinusoidal drive is demonstrated.
See Example 3, page 6 above.
The resistor power loss was reduced by

$$\frac{P_{RQS}}{P_{RQQ}} = 0.902 \qquad \sqrt{\frac{P_{RQS}}{P_{RQQ}}} = 0.9498 \quad \sim \quad \frac{\pi^2}{6\cdot\sqrt{3}}$$

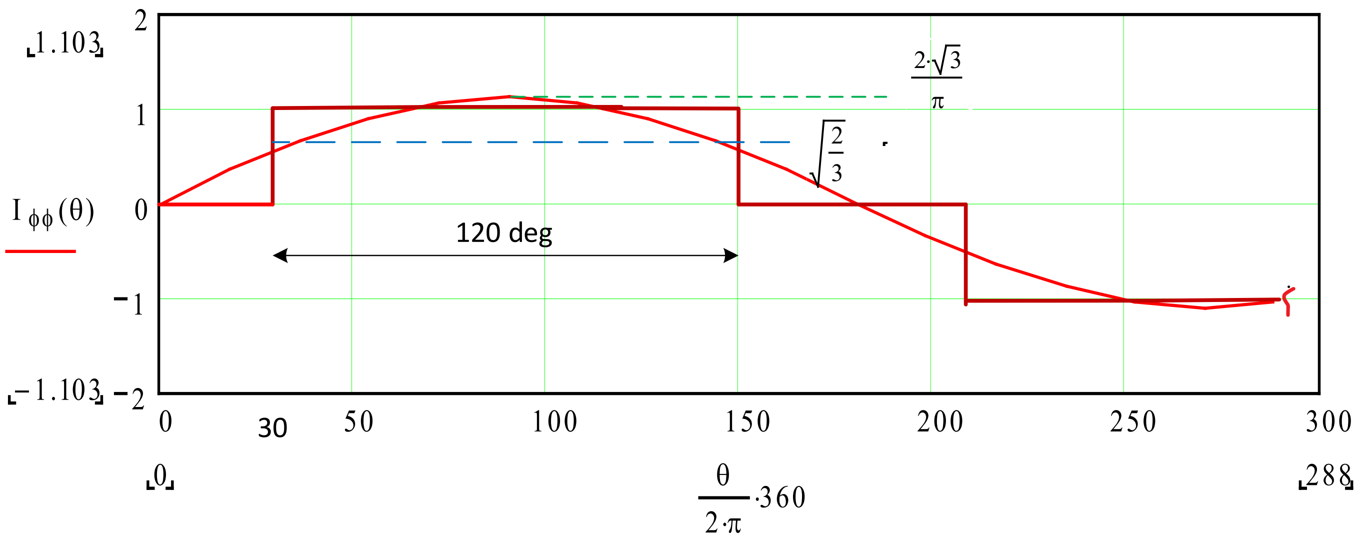
As it is proportional to current squared.
K_{TpQS} is found to be lower than K_{TpQQ} by

$$\left(\frac{3}{\pi}\right)^2 = 0.911891$$

as expected. Everything else in Example 3 looked good.

All results are in agreement with the Equations listed in Attachment A.

Appendix 1 – Squarewave calculation of the Fourier Series peak and total rms value.



The Fourier Series for a pulse wave with no DC is (amplitude +/- 1/2) is given by,

$$\sum_{n=1}^{\infty} \frac{2\cdot A}{n\cdot\pi}\cdot\sin\left(\frac{n\cdot\pi\cdot\tau}{T}\right)\cdot\cos\left(\frac{2\cdot n\cdot\pi}{T}\cdot t - \frac{n\cdot\pi}{T}\cdot\tau\right) \qquad (55)$$

See **Effects of Rise/Fall Times on Signal Spectra by John McClosky, Chief EMC Engineer NASA.** **τ/T = 2/3**

Let F_I be an amplitude multiplier for computing the first harmonic (n=1) then,

$$I_{\phi\phi p}^{sine} = F_I\cdot I_{\phi\phi p} \qquad (56) \qquad F_I = 2\cdot\frac{2}{\pi}\cdot\sin\left(\pi\cdot\frac{2}{3}\right) = \frac{2\cdot\sqrt{3}}{\pi} \qquad (57)$$

Note the doubling of the amplitude (from +/- ½ to +/- 1), i.e. A =2.

And total RMS is found from (SQRT(ratio of conduction period to total period)),

$$I_{\phi\phi rmsQ} = \sqrt{\frac{120\cdot deg}{180\cdot deg}}\cdot I_{\phi\phi p} = \sqrt{\frac{2}{3}}\cdot I_{\phi\phi p} \qquad (58)$$

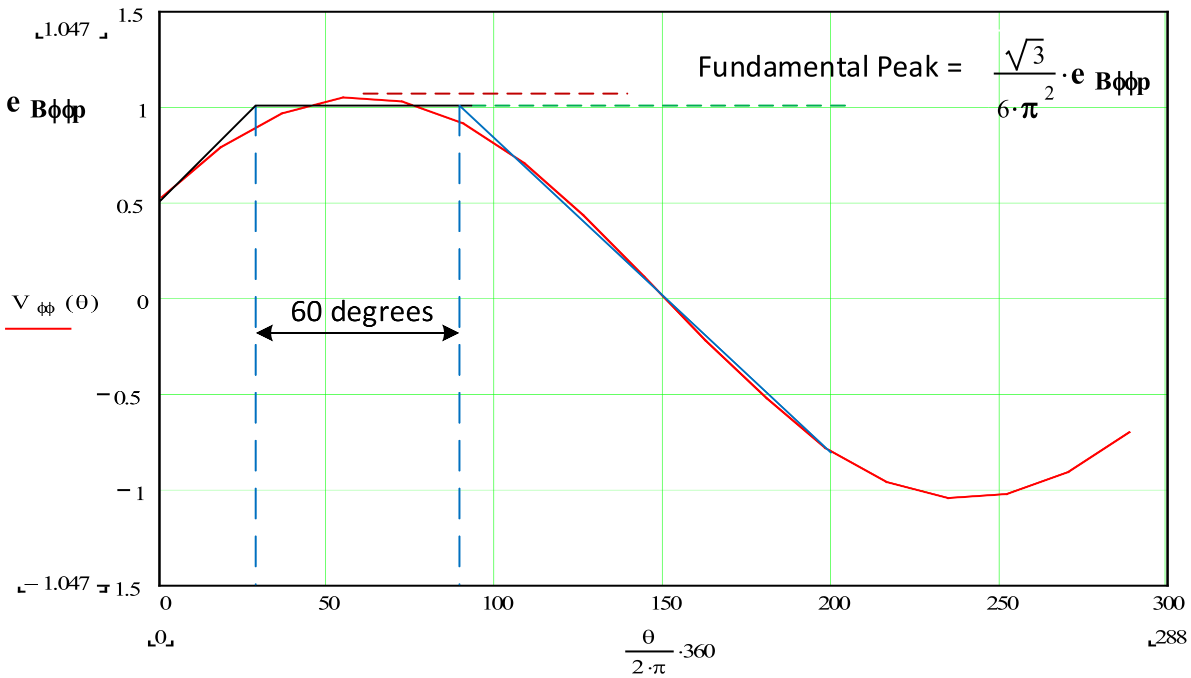
Note that the rms of the fundamental with a peak of $I_{\phi\phi p}$ is found to be

$$\frac{F_I}{\sqrt{2}}\cdot I_{\phi\phi p} = \frac{\sqrt{6}}{\pi}\cdot I_{\phi\phi p} = 0.7796971\cdot I_{\phi\phi p} \qquad (59)$$

Ratio of fundamental rms to the total rms =

$$\frac{I_{\phi\phi rms}}{I_{\phi\phi rmsQ}} = \frac{\sqrt{\frac{6}{\pi}}}{\sqrt{\frac{2}{3}}} = \frac{3}{\pi} = 0.95493 \qquad (60)$$

Appendix 2 – Trapazoidal calculation of the Fourier Series peak and total rms value.



$$F_V = \sqrt{3}\cdot\frac{4\cdot\tau}{T}\cdot\frac{\sin\left(\frac{\pi\cdot\tau}{T}\right)\cdot\sin\left(\frac{\pi\cdot\tau_r}{T}\right)}{\left(\frac{\pi\cdot\tau}{T}\right)\cdot\left(\frac{\pi\cdot\tau_r}{T}\right)} \qquad (61) \qquad \text{Amplitude only}$$

See **Effects of Rise/Fall Times on Signal Spectra by John McClosky, Chief EMC Engineer NASA.**

$$e_{B\phi\phi p}^{\ sine} = F_V\cdot e_{B\phi\phi p} \qquad (62)$$

With

$\tau_r := 1$ $\qquad\qquad$ $\tau_f := \tau_r$

$\tau := 5$

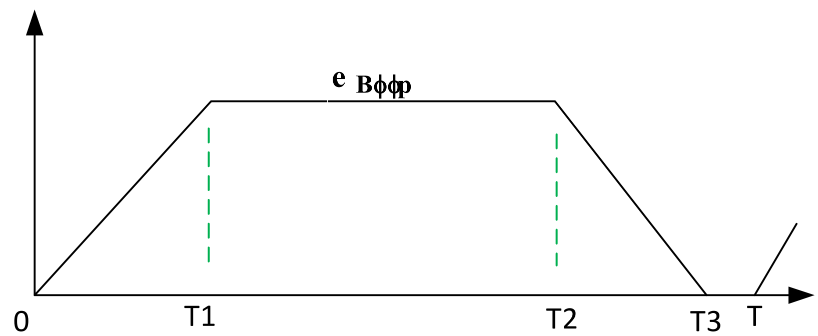
$$T := \tau + \frac{\tau_r + \tau_f}{2}$$

$T = 6$

$$F_V = \sqrt{3}\cdot\frac{6}{\pi^2} = 1.05296 \qquad (63)$$

Calculating a Trapezoidal Waveshapes RMS Value

Waveshape referenced for RMS evaluation:



Note that an adjustment is required to get the proper result.

As the equation below is referenced to 0 then T1, T2 and T3 must have 30 degrees added. Dividing each interval by 60 degrees is done to simplify. And T = T3. *T is 180 degrees in above drawing.*

$T_1 := 1 \qquad T_2 := 2 \qquad T_3 := 3 \qquad T := T_3$

We indicate the trapezoidal waveshapes total rms value by $e_{B\phi\phi rms}{}^{trap}$.

$$e_{B\phi\phi rms}{}^{trap} = e_{B\phi\phi p} \cdot \sqrt{\frac{1}{T} \cdot \left[\left(T_2 - T_1\right) + \frac{\left(T_3 - T_2 + T_1\right)}{3} \right]} \qquad (64)$$

From <u>How to Derive the RMS value of a Trapezoidal waveform – Part 1,</u>
From Mastering Electronics Design Magazine by Adrian S. Nastase, PhD.

https://masteringelectronicsdesign.com/how-to-derive-the-rms-value-of-a-trapezoidal-waveform/

$$e_{B\phi\phi rms}{}^{trap} = \frac{\sqrt{5}}{3} \cdot e_{B\phi\phi p} = 0.74536 \cdot e_{B\phi\phi p} \qquad (65)$$

References:

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2. J.R. Hendershot & T.J.E. Miller, Design of Brushless Permanent-Magnet Machines, Published by Motor Design LLC 2010, pp 405-450.
3. George W. Younkin, P.E. Life Fellow IEEE, *ELECTRIC SERVO MOTOR EQUATIONS AND TIME CONSTANTS,* https://support.controltechnologycorp.com/customer/elearning/younkin/driveMotorEquations.pdf# pp 4-5.
4. John McCloskey, NASA/GSFC Chief EMC Engineer, *Effects of Rise/Fall Times on Signal Spectra*, PowerPoint, Slide 7 and Slide 10.
5. Adrian S. Nastase, PhD., *How to Derive the RMS value of a Trapezoidal waveform – Part 1,* Mastering Electronics Design Magazine. https://masteringelectronicsdesign.com/how-to-derive-the-rms-value-of-a-trapezoidal-waveform/