EXAMPLE 3 "Delta Winding" Square / Square and Square / Sine Evaluation.

Conversion factors:

$ounce\_in := 7.06155181 \cdot 10^{-3} \cdot N \cdot m$ $\quad 1 N m = 141.612\, ounce\_in$ $\quad milli := 10^{-3}$ $\quad \mu := 10^{-6}$

$kg \cdot m^2 = 141.612\, ounce\_in \cdot s^2$ $\quad krpm := 104.72 \cdot \frac{rad}{s}$ $\quad rev := 2 \cdot \pi \cdot rad$

kp = kgf = kilopond force = kg force $\quad 1 \frac{rad}{s} = 0.01\, krpm$ $\quad rpm := \frac{krpm}{1000}$

$kp := 35.274 \cdot \frac{oz}{kg}$ $\quad kgf = 9.80665\, N$ $\quad N = 0.101972\, kgf$

$\frac{2.2046 \cdot lb}{1 \cdot kg} = 1$ $\quad \frac{35.2736 \cdot oz}{1 kg} = 1$ $\quad rms := 1$ $\quad p := 1$ rms and peak abbreviations

$1 \cdot \frac{N \cdot m}{A} = 141.612 \frac{ounce\_in}{A}$ Kt units

$1 \cdot \frac{V}{\frac{rad}{s}} = 104.72 \frac{V}{krpm}$ Ke units

$1 \cdot \frac{ounce\_in}{krpm} = 67.433\, \mu \cdot \frac{N \cdot m}{\frac{rad}{s}}$ D units (mech friction torque)

Motor Example Kollmorgen TBM-6013-A

Datasheet Parameters **[See Vendor Specifications on Page 9]**

The cold specifications are used..

Squarewave specified parameters:

$R_{\phi\phi} := 0.804 \cdot \Omega$ $\quad L_{\phi\phi} := 0.39 \cdot milli \cdot H$ $\quad J_M := 1.41 \cdot 10^{-5} \cdot kg \cdot m^2$ $\quad P := 12$

$K_{TpQQ} := 0.087 \cdot \frac{N \cdot m}{A \cdot p}$ $\quad K_{e\phi\phi p} := 0.087 \cdot \frac{V \cdot p}{\frac{rad}{s}}$ $\quad KTp = K_{e\phi\phi p}$ Checks

$K_{TrmsQQ} := 0.107 \cdot \frac{N \cdot m}{A \cdot rms}$ $\quad K_{TrmsQQ} = 15.152 \frac{ounce\_in}{A \cdot rms}$ Checks

$K_{e\phi\phi rmsQQ} := 6.44 \cdot \frac{V \cdot rms}{krpm}$ $\quad K_{e\phi\phi rmsQQ} = 0.061 \frac{V \cdot rms}{\frac{rad}{s}}$?? assumes a $\frac{1}{\sqrt{2}}$ conversion

Which is Not true.

$K_{MQQ} := 0.097 \cdot \frac{N \cdot m}{\sqrt{W}}$

Conversions to SI Units

Verification Square / Square row

$$K_{TrmsQQ} = \sqrt{\frac{3}{2}}\cdot K_{TpQQ} = 0.10655\,\frac{N\cdot m}{A\cdot rms}$$

Attachment A, Chart 1 Col. 2

$$K_{TrmsQQ} = \sqrt{\frac{3}{2}}\cdot K_{e\phi\phi p} = 0.10655\,\frac{N\cdot m}{A\cdot rms}$$

Attachment A, Chart 1 (Col. 1 x Col 2)

$$\frac{K_{TpQQ}}{K_{e\phi\phi p}} = 1$$

Attachment A, Chart 1 Col 1

$$K_{e\phi\phi rmsQQ} := \frac{\sqrt{5}}{3}\cdot K_{e\phi\phi p} \qquad K_{e\phi\phi rmsQQ} = 0.065\,\frac{V\cdot rms}{\frac{rad}{s}}$$

Attachment A, Chart 1 Col 4
This is the correct value!

$$K_{e\phi\phi rmsQQ} := \frac{1}{\sqrt{3}}\cdot\sqrt{\frac{10}{9}}\cdot K_{TrmsQQ}$$

Attachment A, Chart 1 Col 3

$$K_{e\phi\phi rmsQQ} = 0.065\,\frac{V\cdot rms}{\frac{rad}{s}}$$

Figures of Merit - Calculated

Stall Torque

$V_{\phi\phi ptest} := 3.83526\cdot V\cdot p$ Solved for $V\phi\phi p$ from Spec.

From (101) "Page 14" $\tau_{stallQQ} := \frac{V_{\phi\phi ptest}}{R_{\phi\phi}}\cdot K_{TpQQ}$

(41) $I_{\phi\phi pstallQQ} := \frac{V_{\phi\phi ptest}}{R_{\phi\phi}}$ $I_{\phi\phi pstallQQ} = 4.77\,A\,p$

$\tau_{stallQQ} = 0.415\,N\cdot m$ $\tau_{stallQQ} = 58.77\,\mathrm{ounce\_in}$

$I_{\phi\phi rmsstallQQ} := \frac{\tau_{stallQQ}}{K_{TrmsQQ}}$ $I_{\phi\phi rmsstallQQ} = 3.879\,A\cdot rms$

$$\frac{I_{\phi\phi rmsstallQQ}}{I_{\phi\phi pstallQQ}} = 0.813 \quad \sim \quad \sqrt{\frac{2}{3}}$$

Attachment A (A5)

$V_{\phi\phi ptest1} := 12.85\cdot V\cdot p$

$\tau_{stallPeak} := \frac{V_{\phi\phi ptest1}}{R_{\phi\phi}}\cdot K_{TpQQ}$ $\tau_{stallPeak} = 1.39\,N\cdot m$ Checks

$I_{\phi\phi stallPeakQQ} := \frac{V_{\phi\phi ptest1}}{R_{\phi\phi}}$ $I_{\phi\phi stallPeakQQ} = 15.983\,A\cdot p$ @ 25 deg C

No load speed @ 2.856 Vrms

$V_{\phi\phi rmstest} := V_{\phi\phi ptest}\cdot\frac{\sqrt{5}}{3}\cdot\frac{rms}{p}$

Using (42) $\omega_{NL1} := \frac{V_{\phi\phi ptest}}{K_{e\phi\phi p}}$ $\omega_{NL1} = 44.083\,\frac{rad}{s}$

$V_{\phi\phi rmstest} = 2.859\,V$

$\omega_{NL1} := \frac{V_{\phi\phi rmstest}}{K_{e\phi\phi rmsQQ}}$ $\omega_{NL1} = 43.899\,\frac{rad}{s}$ ~ Match

Speed -Torque Curve

From (45) $R_{mQQ} := \dfrac{R_{\phi\phi}}{K_{e\phi\phi p}\cdot K_{TpQQ}}$ $R_{mQQ} = 106.223\,\dfrac{\frac{rad}{s}}{N\cdot m}$

$= \dfrac{3}{2}\cdot\dfrac{R_{\phi\phi}}{K_{TrmsQQ}{}^{2}} = 105.337\,\dfrac{\frac{rad}{s}}{N\cdot m}$ close enough Attachment A, Chart 1 Inverse (Col 5)

Speed-Torque test points

Torque test values:

$\tau_1 := 0\cdot ounce\_in$ $\tau_1 = 0\,N\cdot m$

$\tau_2 := 15\cdot ounce\_in$ $\tau_2 = 0.106\,N\cdot m$

$\tau_3 := 40\cdot ounce\_in$ $\tau_3 = 0.282\,N\cdot m$

Using (46)

$\omega_1 := \omega_{NL1} - R_{mQQ}\cdot\tau_1$ $\omega_1 = 43.9\,\dfrac{rad}{s}$

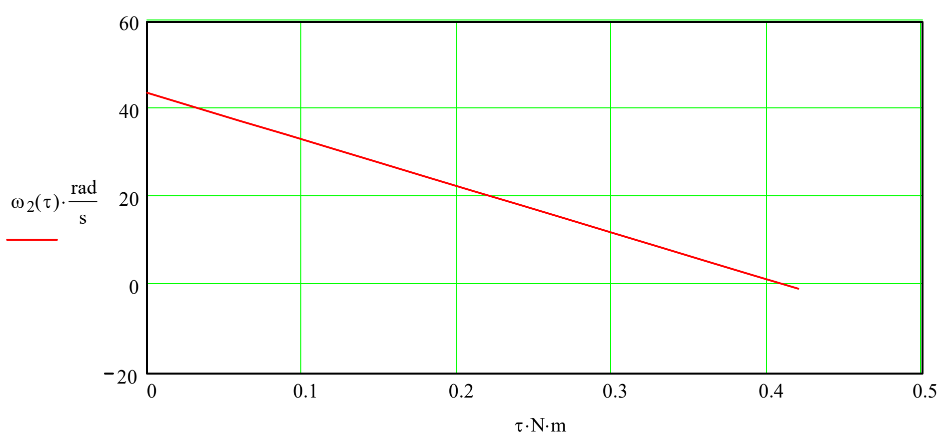
$\omega_2 := \omega_{NL1} - R_{mQQ}\cdot\tau_2$ $\omega_2 = 32.65\,\dfrac{rad}{s}$

$\omega_3 := \omega_{NL1} - R_{mQQ}\cdot\tau_3$ $\omega_3 = 13.9\,\dfrac{rad}{s}$

$\tau := 0\cdot N\cdot m, (0.03\cdot N\cdot m)\,..\,0.42\cdot N\cdot m$ Define range variable

Plot ω vs τ

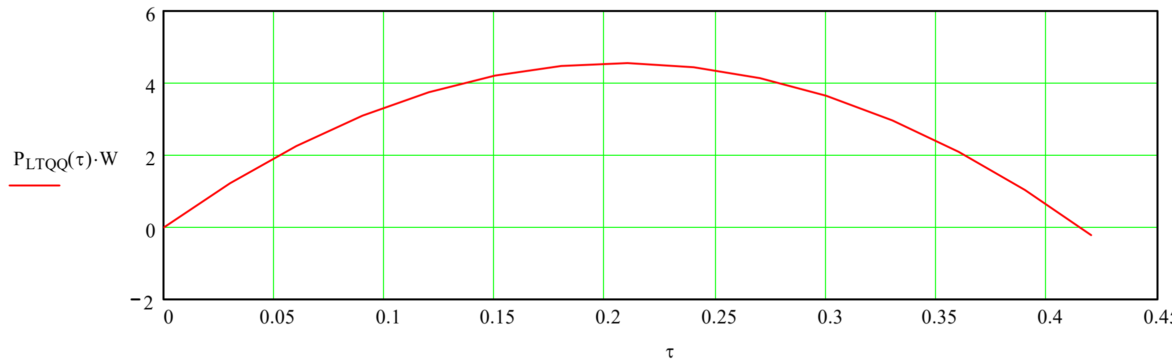
Speed Torque Curve $\omega_2(\tau) := \omega_{NL1} - R_{mQQ}\cdot\tau$ Equation (46)



$\omega_2(0\cdot N\cdot m) = 43.899\,\dfrac{rad}{s}$ $\omega_2\left(\dfrac{\tau_{stallQQ}}{2}\right) = 21.857\,\dfrac{rad}{s}$ $\omega_2(\tau_{stallQQ}) = -0\,\dfrac{rad}{s}$

Load Power as a function of torque:

$$P_{LTQQ}(\tau) := -\left(\frac{\omega_{NL1}}{\tau_{stallQQ}}\right)\cdot\tau^2 + \omega_{NL1}\cdot\tau$$



ω*τ

$$P_{LTQQ}\left(\frac{\tau_{stallQQ}}{2}\right) = 4.555\,W \qquad \frac{\tau_{stallQQ}}{2}\cdot\frac{\omega_{NL1}}{2} = 4.555\,W$$

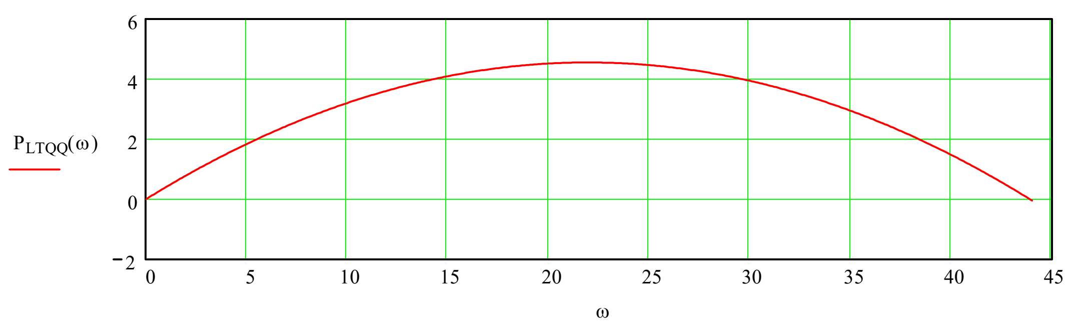
Checks

$$\omega := 0\cdot\frac{rad}{s}, \left(0.1\cdot\frac{rad}{s}\right) .. 44\cdot\frac{rad}{s}$$

Define range variable

Load Power as a function of speed:

$$P_{LTQQ}(\omega) := -\left(\frac{\tau_{stallQQ}}{\omega_{NL1}}\right)\cdot\omega^2 + \tau_{stallQQ}\cdot\omega$$



Using (30)

$$P_{RQQ} := \left(\frac{\frac{\tau_{stallQQ}}{2}}{K_{TpQQ}}\right)^2\cdot R_{\phi\phi}$$

with argument for $I\phi\phi$

$$I_{\phi\phi pstall2QQ} := \frac{\frac{\tau_{stallQQ}}{2}}{K_{TpQQ}}$$

$$P_{RQQ} = 4.574\,W$$

$$I_{\phi\phi pstall2QQ} = 2.385\,A\cdot p$$

Using (23) + (24)

$$P_{LTQQ} := 4.5546\cdot W \qquad P_{RQQ} = 4.574\,W$$

$$P_T := \frac{\omega_{NL1}}{2}\cdot\frac{\tau_{stallQQ}}{2} + R_{\phi\phi}\cdot\left(\frac{\frac{\tau_{stallQQ}}{2}}{K_{TpQQ}}\right)^2$$

$$P_T = 9.128\,W \qquad P_{LTQQ} + P_{RQQ} = 9.128\,W$$

Checks

Using (49)

$$K_{MQQ} := \frac{K_{TpQQ}}{\sqrt{R_{\phi\phi}}} \qquad K_{MQQ} = 0.097\,\frac{N\cdot m}{\sqrt{W}}$$ Checks $$K_{MQQ} = 13.74\,\frac{ounce\_in}{\sqrt{W}}$$ Checks

$$K_{MQQ} := \sqrt{\frac{2}{3}}\cdot\frac{K_{TrmsQQ}}{\sqrt{R_{\phi\phi}}} \qquad K_{MQQ} = 0.097\,\frac{N\cdot m}{\sqrt{W}}$$ Using Attachment A, Chart 2 Col 7

Going forward $$K_{MQQ} := 0.097\cdot\frac{N\cdot m}{\sqrt{W}}$$

Using (53)

$$\tau_{mQQ} := \frac{R_{\phi\phi}\cdot J_M}{K_{e\phi\phi p}\cdot K_{TpQQ}}$$

$$\tau_{mQQ} = 1.498\,milli\cdot s$$

Using (54)

$$\tau_e := \frac{L_{\phi\phi}}{R_{\phi\phi}}$$

No published values

$$\tau_e = 0.485\,milli\cdot s$$

Find $$K_{TpQQ}\cdot K_{e\phi\phi p} = 0.008\,\Omega\,\frac{N\cdot m}{\frac{rad}{s}}$$

From Attachment A Chart 1 Col. 5

$$\frac{2}{3}K_{TrmsQQ}{}^2 = 0.008\,\Omega\cdot\frac{N\cdot m}{\frac{rad}{s}}$$

From Attachment A Chart 2 Inverse [Col 10]

$$\sqrt{\frac{6}{5}}\,K_{TrmsQQ}\cdot K_{e\phi\phi rmsQQ} = 0.008\,\Omega\,\frac{N\cdot m}{\frac{rad}{s}}$$

Both Check.

Resistor power calculation using the Motor Constant, KM,

$$P_R = \left(\frac{\tau}{K_M}\right)^2$$ Derived from definition (102) "Page 15"

$$P_{RQQ} := \left(\frac{\frac{\tau_{stallQQ}}{2}}{K_{MQQ}}\right)^2 \qquad P_{RQQ} = 4.576\,W$$ Checks

$$P_{RQQ} = R_{\phi\phi}\cdot\left(\frac{\frac{\tau_{stallQQ}}{2}}{K_{TpQQ}}\right)^2 = 4.574\,W$$

$$= \frac{3}{2}\cdot R_{\phi\phi}\cdot\left(\frac{\frac{\tau_{stallQQ}}{2}}{K_{TrmsQQ}}\right)^2 = 4.574\cdot W$$

Load power equation verifications. $P_{LTQQ} = \frac{\omega_{NL1}}{2}\cdot\left(\frac{\tau_{stallQQ}}{2}\right) = 4.555\,W$ $(\tau\cdot\omega)$

From (28a)

$$P_{LTQQ} := K_{e\phi\phi p}\cdot\left(\frac{\omega_{NL1}}{2}\right)\cdot\left(\frac{\frac{\tau_{stallQQ}}{2}}{K_{TpQQ}}\right)$$

with Iϕϕp calculation $I_{\phi\phi pQQ} := \frac{\frac{\tau_{stallQQ}}{2}}{K_{TpQQ}}$

$P_{LTQQ} = 4.555\,W$ Acceptable Check $I_{\phi\phi pQQ} = 2.385\,A\cdot p$

From (28b)

$$P_{LTQQ} := \sqrt{3}\cdot\sqrt{\frac{9}{10}}\cdot K_{e\phi\phi rmsQQ}\cdot\left(\frac{\omega_{NL1}}{2}\right)\cdot\left(\frac{\frac{\tau_{stallQQ}}{2}}{K_{TrmsQQ}}\right)$$

with Iϕϕrms calculation $I_{\phi\phi rmsQQ} := \frac{\frac{\tau_{stallQQ}}{2}}{K_{TrmsQQ}}$

$I_{\phi\phi rmsQQ} = 1.939\,A\cdot rms$

$P_{LTQQ} = 4.555\,W$ Acceptable Check $\frac{I_{\phi\phi rmsQQ}}{I_{\phi\phi pQQ}} \sim \sqrt{\frac{2}{3}}$ (A5) OK < 0.5%

Sinewave Drive & Calculations:

$K_{e\phi\phi p} := 0.087\cdot\frac{V\cdot p}{\frac{rad}{s}}$ $K_{e\phi\phi p} = 12.32\,\frac{ounce\_in}{A\cdot p}$ Keϕϕp is invariant for drive

From (A22)

Verification Square / Sine row

$K_{e\phi\phi rmsQS} := \left(\frac{3}{\pi}\right)^2\cdot\sqrt{\frac{2}{3}}\cdot K_{e\phi\phi p}$ $K_{e\phi\phi rmsQS} = 0.065\,\frac{V\cdot rms}{\frac{rad}{s}}$ Attachment A, Chart 1, Col. 4.

$K_{TpQS} := \sqrt{\frac{3}{2}}\cdot K_{e\phi\phi rmsQS}$ $K_{TpQS} = 0.079\,\frac{N\cdot m}{A\cdot p}$ Attachment A, Chart 1 INV[Col 2 x Col 3]

INV = Inverse or reciprocal.

$K_{TpQS} := \left(\frac{3}{\pi}\right)^2\cdot K_{e\phi\phi p}$ $K_{TpQS} = 0.079\,\frac{N\cdot m}{A\cdot p}$ Attachment A, Chart 1 Col 2

$K_{TrmsQS} := \sqrt{2}\cdot K_{TpQS}$ $K_{TrmsQS} = 0.112\,\frac{N\cdot m}{A\cdot rms}$ Attachment A, Chart 1 Col 1

$K_{TrmsQS} := \sqrt{3}\cdot K_{e\phi\phi rmsQS}$ $K_{TrmsQS} = 0.112\,\frac{N\cdot m}{A\cdot rms}$ Attachment A, Chart 1 INV[Col 3]

$K_{TrmsQS} := \left(\frac{3}{\pi}\right)^2\cdot\sqrt{2}\cdot K_{e\phi\phi p}$ $K_{TrmsQS} = 0.112\,\frac{N\cdot m}{A\cdot rms}$ Attachment A, [Col 1 x Col 2]

$K_{TrmsQQ} := 0.10655\cdot\frac{N\cdot m}{A\cdot rms}$ $K_{TrmsQS} = 0.1122\,\frac{N\cdot m}{A\cdot rms}$

$\frac{K_{TrmsQS}}{K_{TrmsQQ}} = 1.053$ $\frac{6}{\pi^2}\cdot\sqrt{3} = 1.053$ Checks (A30)

$V_{\phi\phi ptest} = 3.835\,V\cdot p$

From (48) $$K_{MQS} := \sqrt{\frac{2}{3}}\cdot\frac{K_{TrmsQS}}{\sqrt{R_{\phi\phi}}}$$ $$K_{MQS} = 14.468\,\frac{ounce\_in}{\sqrt{W}}$$ $$K_{MQS} = 0.102\,\frac{N\cdot m}{\sqrt{W}}$$

Note the increase in Motor Constant due to increase in efficiency.

Note also that

$$K_{MQS} := \frac{2}{\sqrt{3}}\cdot\frac{K_{TpQS}}{\sqrt{R_{\phi\phi}}} \qquad K_{MQS} = 14.468\,\frac{ounce\_in}{\sqrt{W}}$$

Attachment A, Chart 1, Col 2 relationship.

Using (100) "Page 13"

$$\tau_{stallQS} := \frac{2}{\sqrt{3}}\cdot\frac{V_{\phi\phi ptest}}{R_{\phi\phi}}\cdot K_{TpQS} \qquad \tau_{stallQS} = 0.437\,N\cdot m$$

Stall torque increases

From (42)

$$\omega_{NL} := \frac{V_{\phi\phi ptest}}{K_{e\phi\phi p}} \qquad \omega_{NL} = 44.083\,\frac{rad}{s}$$

NO Change

Using (44) $$R_{mQS} := \frac{\sqrt{3}}{2}\cdot\left(\frac{R_{\phi\phi}}{K_{TpQS}\cdot K_{e\phi\phi p}}\right) \qquad R_{mQS} = 100.88\,\frac{\frac{rad}{s}}{N\cdot m}$$

Slope decreased

Note we want to use the QQ vallues for stall torque / 2 and no load speed / 2 so a direct comparison can be made.

As RmQQ > RmQS we adjust Vϕϕp so that the two speed torque curves intersect at this point.

$$\tau_x := \frac{\tau_{stallQQ}}{2} \qquad \tau_x = 0.208\,N\cdot m \qquad \omega_x := \frac{\omega_{NL1}}{2} \qquad \omega_x = 21.95\,\frac{rad}{s}$$

Using (46) $$\omega_{NLx} := \omega_x + R_{mQS}\cdot\tau_x \qquad \omega_{NLx} = 42.883\,\frac{rad}{s}$$

$$V_{\phi\phi ptestQS} := \omega_{NLx}\cdot K_{e\phi\phi p} \qquad V_{\phi\phi ptestQS} = 3.731\,V\cdot p$$

To match QQ load power

Resistor power with sinewave drive.

Using (38) $$P_{RQS} := \frac{3}{2}\cdot R_{\phi\phi}\cdot\left(\frac{\tau_x}{K_{TrmsQS}}\right)^2 \qquad P_{RQS} = 4.125\,W$$

with $P_{RQQ} = 4.576\,W$

$$\frac{P_{RQQ}}{P_{RQS}} = 1.109 \qquad \sqrt{\frac{P_{RQQ}}{P_{RQS}}} = 1.053 \quad \sim \quad \frac{6\cdot\sqrt{3}}{\pi^2} = 1.053$$

Checks with (A30)

Load power check

(36a)

$$P_{LTQS} := \left(\frac{3}{\pi}\right)^2 \cdot K_{e\phi\phi p} \cdot \omega_x \cdot \left(\frac{\tau_x}{K_{TpQS}}\right)$$

$P_{LTQS} = 4.555\,W$

(36b)

$$P_{LTQS} := \sqrt{3} \cdot K_{e\phi\phi rmsQS} \cdot \omega_x \cdot \left(\frac{\tau_x}{K_{TrmsQS}}\right)$$

$P_{LTQS} = 4.555\,W$

$$I_{\phi\phi pQS} := \frac{\tau_x}{K_{TpQS}}$$

$I_{\phi\phi pQS} = 2.616\,A \cdot p$

$$I_{\phi\phi rmsQS} := \frac{\tau_x}{K_{TrmsQS}}$$

$I_{\phi\phi rmsQS} = 1.849\,A \cdot rms$

$$\frac{I_{\phi\phi pQS}}{I_{\phi\phi rmsQS}} = \sqrt{2} \qquad \text{(A23)}$$

TBM(S) 60 Series Performance Data and Motor Parameters

Motor Parameter	Symbol	Units	TOL	TBM(S)-6013-X A	TBM(S)-6013-X B
Continuous Stall Torque*	Tc	N-m	NOM	0.415	0.390
		oz-in		58.8	55.2
Continuous Current	Ic	Adc	NOM	5.51	8.30
		Arms		4.50	6.78
Peak Stall Torque* (25°C winding temp)	Tp	N-m	NOM	1.39	1.23
		oz-in		197	174
Peak Current	Ip	Adc	NOM	19.0	26.9
		Arms		15.5	22.0
Rated Cont Power*	P Rated	Watts	NOM	114	103
Speed at Rated Power	N Rated	RPM	NOM	4000	3540
Design Voltage	Vbus	Vdc	NOM	48.0	24.0
	Vac	Vrms	NOM	33.9	17.0
Torque Sensitivity at Temp*	Kt (hot)	N-m / Adc	+/-10%	0.079	0.049
		oz-in / Adc		11.2	7.00
		N-m / Arms	+/-10%	0.097	0.061
		oz-in / Arms		13.7	8.57
Back EMF at Temp*	Kb (hot)	Vpk / kRPM	+/-10%	8.28	5.18
		Vrms / kRPM		5.86	3.66
Torque Sensitivity at 25°C	Kt (cold)	N-m / Adc	+/-10%	0.087	0.054
		oz-in / Adc		12.3	7.70
		N-m / Arms	+/-10%	0.107	0.067
		oz-in / Arms		15.1	9.43
Back EMF	Kb (cold)	Vpk / kRPM	+/-10%	9.11	5.69
		Vrms/kRPM		6.44	4.03
Motor Constant at 25°C	Km (cold)	N-m/√watt	+/-10%	0.097	0.091
		oz-in/√watt		13.7	12.9
Motor Constant at Temp	Km (hot)	N-m/√watt	+/-10%	0.072	0.068
		oz-in/√watt		10.2	9.59
Resistance at 25°C	Rm	Ohms	+/- 10%	0.804	0.355
Inductance	Lm	mH	+/- 30%	0.39	0.15
Inertia*	Jm	Kg-m²		1.41E-05	
		oz-in-s²		2.00E-03	
Weight*	Wt	grams		221	
		oz		7.77	
Max Static Friction	Tf	N-m		0.021	
		oz-in		2.93	
Cogging Friction (Peak-to-Peak)	Tcog	N-m		0.009	
		oz-in		1.22	
Viscous Damping	Fi	N-m/ kRPM		2.04E-03	
		oz-in / kRPM		2.88E-01	
Thermal Resistance*	TPR	°C / watt		3.55	
Number of Poles	P	-		12	

*Notes
1) Continuous Stall Torque and Rated Power assume ambient temperature of 25°C
2) Winding temp = 155°C for Kt and Kb hot
3) Inertia and weight assume max thru-bore
4) TPR assumes motor is housed and mounted to a 3.5" x 3.5" x 0.25" heat sink or equivalent
5) Some Peak and Continuous Torques may be limited by lead wire gauge