EXAMPLE 2 "WYE Winding -- Sine / Sine Evaluation"

Conversion factors:

$\text{ounce\_in} := 7.06155181 \cdot 10^{-3} \cdot N \cdot m$ $\quad 1Nm = 141.612\,\text{ounce\_in}$ $\quad \text{milli} := 10^{-3}$ $\quad \mu := 10^{-6}$

$kg \cdot m^2 = 141.612\,\text{ounce\_in} \cdot s^2$ $\quad \text{krpm} := 104.72 \cdot \frac{rad}{s}$ $\quad \text{rev} := 2 \cdot \pi \cdot rad$

kp = kgf = kilopond force = kg force $\quad 1\frac{rad}{s} = 9.549 \times 10^{-3}\,\text{krpm}$ $\quad \text{rpm} := \frac{\text{krpm}}{1000}$

$kp := 35.274 \cdot \frac{oz}{kg}$ $\quad kgf = 9.80665\,N$ $\quad N = 0.101972\,kgf$

$\frac{2.2046 \cdot lb}{1 \cdot kg} = 1$ $\quad \frac{35.2736 \cdot oz}{1kg} = 1$ $\quad rms := 1$ $\quad p := 1$ rms and peak abbreviations

$1 \cdot \frac{N \cdot m}{A} = 141.612\,\frac{\text{ounce\_in}}{A}$ Kt units

$1 \cdot \frac{V}{\frac{rad}{s}} = 104.72\,\frac{V}{\text{krpm}}$ Ke units

$1 \cdot \frac{\text{ounce\_in}}{\text{krpm}} = 6.743 \times 10^{-5}\,\frac{N \cdot m}{\frac{rad}{s}}$ D units (mech friction torque)

Motor Example BEI DIP30-07-001A

Datasheet Parameters **[See Motor Specifications on Page 6]**

This poses a set of problems due to the fact that the KT and Ke values are ambiguous with respect to pk or rms. Also listed is Kt = Ke (0.226 V / rad / s = 32 ounce_in / A). so we need to identify the proper units.

Sinusoidal Specifications

$R_{\phi\phi} := 5.6 \cdot \Omega$ $\quad P := 12$ No. of magnetic poles

$K_M := 13.5 \cdot \frac{\text{ounce\_in}}{\sqrt{W}}$

$L_{\phi\phi} := 8 \cdot \text{milli} \cdot H$ $\quad J_M := 0.001923 \cdot \text{ounce\_in} \cdot s^2$

$\tau_m := 1.49 \cdot \text{milli} \cdot s$ $\quad \tau_3 := 1.43 \cdot \text{milli} \cdot s$

$\omega_0 := 106 \cdot \frac{rad}{s}$ Theoretical no load speed @ 24 V.

$\tau_p := 100 \cdot \text{ounce\_in}$ Peak (stall) torque.

Using (48), solving for KTrms $$K_{TrmsSS} := K_M \cdot \sqrt{\frac{3}{2}} \cdot \sqrt{R_{\phi\phi}}$$

$$K_{TrmsSS} = 0.276 \frac{N \cdot m}{A \cdot rms} \qquad K_{TrmsSS} = 39.127 \frac{ounce\_in}{A \cdot rms}$$

Continuing with the parameter relationships:

and converting to SI Units

Verification Sine / Sine row

$$K_{TpSS} := \frac{K_{TrmsSS}}{\sqrt{2}}$$

Attachment A, Chart 1, Col 2

$$K_{TpSS} = 0.195\,N \cdot \frac{m}{A \cdot p}$$

$$K_{e\phi\phi p} := \sqrt{\frac{2}{3}} \cdot K_{TrmsSS}$$

Attachment A, Chart 1, INV(Col 1*Col. 2)

$$K_{e\phi\phi p} = 0.226 \frac{V}{\frac{rad}{s}}$$

So the Keϕϕp appears to be the correct value for back emf.

$$K_{TpSS} := \frac{\sqrt{3}}{2} \cdot K_{e\phi\phi p}$$

Attachment A, Chart 1, Col 1

$$K_{TpSS} = 0.195 \frac{N \cdot m}{A \cdot p}$$

$$K_{e\phi\phi rmsSS} := \frac{K_{TrmsSS}}{\sqrt{3}}$$

Attachment A, Chart 1, Col 3

$$K_{e\phi\phi rmsSS} = 0.16 \frac{V \cdot rms}{\frac{rad}{s}}$$

$$K_{e\phi\phi p} := \sqrt{2} \cdot K_{e\phi\phi rmsSS}$$

Attachment A, Chart 1, Col 4

$$K_{e\phi\phi p} = 0.226 \frac{V \cdot p}{\frac{rad}{s}} \qquad K_{MSS} := K_M$$

$$J_M = 1.358 \times 10^{-5}\,kg\,m^2 \qquad K_{MSS} = 0.0953 \frac{N \cdot m}{\sqrt{W}}$$

$$K_{e\phi\phi pSS} := K_{e\phi\phi p}$$

Though Keϕϕp is mostly used.

Figures of Merit - Calculated

Stall Torque

$V_{\phi\phi ptest} := 17.4V \cdot p$

(100)
"Page 13"

$\tau_{stallSS} := \frac{2}{\sqrt{3}} \cdot \frac{V_{\phi\phi ptest}}{R_{\phi\phi}} \cdot K_{TpSS}$

From (40) $I_{\phi\phi pstall} := \frac{2}{\sqrt{3}} \cdot \frac{V_{\phi\phi ptest}}{R_{\phi\phi}}$ $I_{\phi\phi pstall} = 3.588\,A$

$I_{\phi\phi pstall\_spec} = \frac{V_{\phi\phi ptest}}{R_{\phi\phi}} = 3.107 \cdot A \cdot p$

Incorrect equation (41) used in Spec.! Use for Squarewave drive only.

$I_{\phi\phi rmsstall} := \frac{I_{\phi\phi pstall}}{\sqrt{2}}$ $I_{\phi\phi rmsstall} = 2.537\,A \cdot rms$

$\tau_{stallSS} = 0.701\,N \cdot m$ $\tau_{stallSS} = 99.263\,ounce\_in$

Check
Spec states 100 ounce_in for Tp

No load speed @ 24 V

$V_{\phi\phi ptest} := 24 \cdot V \cdot p$

Using (42) $\omega_{NL} := \frac{V_{\phi\phi ptest}}{K_{e\phi\phi p}}$ $\omega_{NL} = 106.386 \frac{rad}{s}$ Checks with spec. (ω0)

Speed-Torque Curve

(42) $\omega_{NL} = \frac{V_{\phi\phi p}}{K_{e\phi\phi p}} = \frac{V_{\phi\phi rms}}{K_{e\phi\phi rms}}$

No load test values:

$V_{\phi\phi p} := 17.4 \cdot V \cdot p$ $\omega_{NL1} := \frac{V_{\phi\phi p}}{K_{e\phi\phi p}}$ (42) $\omega_{NL1} = 77.13 \frac{rad}{s}$

$V_{\phi\phi rms1} := V_{\phi\phi p} \cdot \frac{\frac{rms}{p}}{\sqrt{2}}$ $\omega_{NL1} := \frac{V_{\phi\phi rms1}}{K_{e\phi\phi rmsSS}}$ $\omega_{NL1} = 77.13 \frac{rad}{s}$ Checks

Torque test values

$\tau_1 := 0 \cdot ounce\_in$ $\tau_1 = 0\,N \cdot m$

$\tau_2 := 35.00 \cdot ounce\_in$ $\tau_2 = 0.247\,N \cdot m$

$\tau_3 := 90 \cdot ounce\_in$ $\tau_3 = 0.636\,N \cdot m$

Speed − Torque slope

Using (44) $R_{mSS} := \frac{\sqrt{3}}{2} \cdot \frac{R_{\phi\phi}}{K_{e\phi\phi rmsSS} \cdot K_{TrmsSS}}$ $R_{mSS} = 110.035 \frac{\frac{rad}{s}}{N \cdot m}$

Speed-Torque test points

Using (46)

$\omega_1 := \omega_{NL1} - R_{mSS} \cdot \tau_1$ $\qquad \omega_1 = 77.13 \, \frac{rad}{s}$

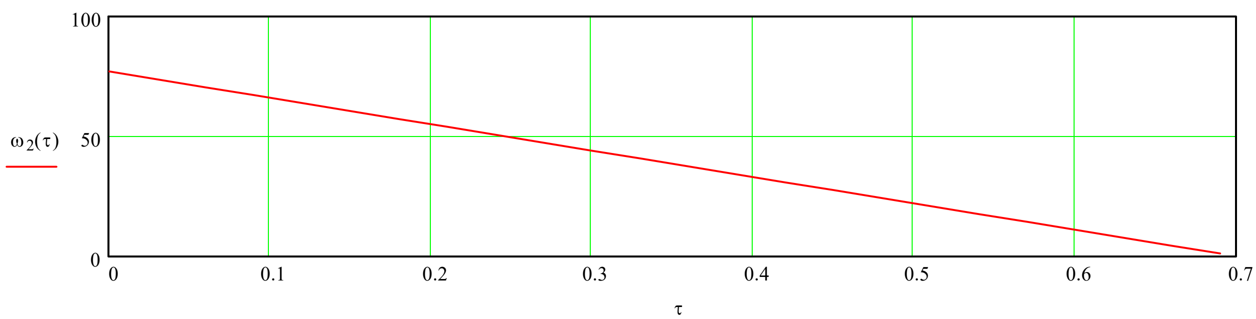
$\omega_2 := \omega_{NL1} - R_{mSS} \cdot \tau_2$ $\qquad \omega_2 = 49.934 \, \frac{rad}{s}$

$\omega_3 := \omega_{NL1} - R_{mSS} \cdot \tau_3$ $\qquad \omega_3 = 7.198 \, \frac{rad}{s}$

$\tau := 0 \cdot N \cdot m, (0.03 \cdot N \cdot m) .. 0.7 \cdot N \cdot m$ $\qquad$ define range variable

Plot ω2 vs τ

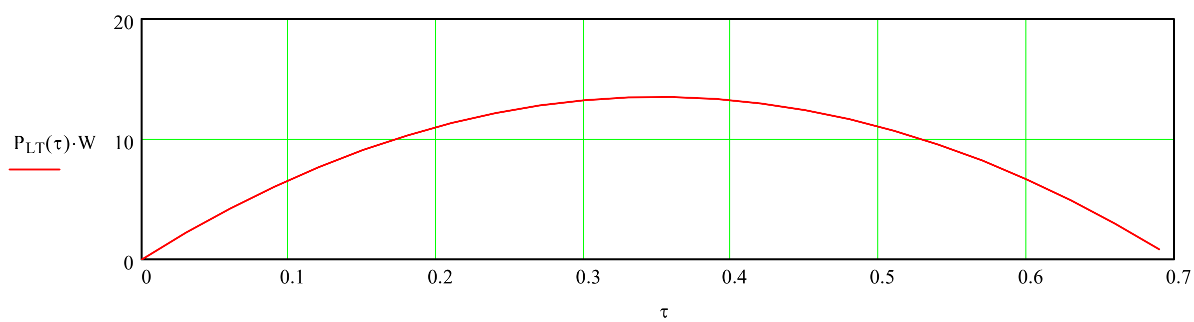
Speed Torque Curve $\qquad \omega_2(\tau) := \omega_{NL1} - R_{mSS} \cdot \tau$ $\qquad$ Equation (46)



$\omega_2(0 \cdot N \cdot m) = 77.13 \, \frac{rad}{s}$ $\qquad \omega_2\left(\frac{\tau_{stallSS}}{2}\right) = 38.565 \, \frac{rad}{s}$ $\qquad \omega_2(\tau_{stallSS}) = -0 \, \frac{rad}{s}$

Load Power as a function of torque:

$$P_{LT}(\tau) := -\left(\frac{\omega_{NL1}}{\tau_{stallSS}}\right) \cdot \tau^2 + \omega_{NL1} \cdot \tau$$

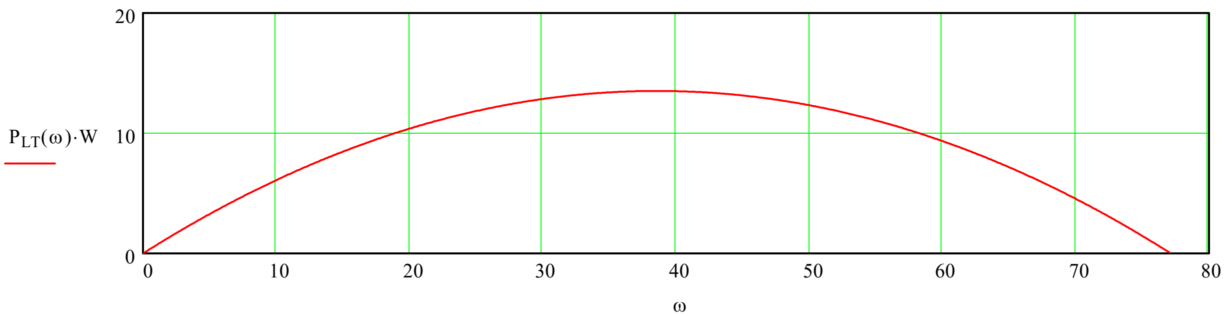


$$P_{LT}\left(\frac{\tau_{stallSS}}{2}\right) = 13.516 \, W$$

$$\omega := 0\cdot\frac{rad}{s}, \left(0.1\cdot\frac{rad}{s}\right) .. 77\cdot\frac{rad}{s}$$ Range variable defined

Load Power as a function of speed

$$P_{LT}(\omega) := -\left(\frac{\tau_{stallSS}}{\omega_{NL1}}\right)\cdot\omega^2 + \tau_{stallSS}\cdot\omega$$



$$P_{LT}\left(\frac{\omega_{NL1}}{2}\right) = 13.516\,W$$

Using (27)

$$P_R := \frac{3}{2}\cdot\left(\frac{\frac{\tau_{stallSS}}{2}}{K_{TrmsSS}}\right)^2\cdot R_{\phi\phi}$$ with $I\phi\phi$ argument

$$I_{\phi\phi rmsStall} := \left(\frac{\tau_{stallSS}}{K_{TrmsSS}}\right)$$

$$P_R = 13.516\,W$$

$$I_{\phi\phi rmsStall} = 2.537\,A$$

Using (14)+(15)

$$P_T := \frac{\omega_{NL1}}{2}\cdot\frac{\tau_{stallSS}}{2} + \frac{3}{2}\cdot R_{\phi\phi}\cdot\left(\frac{\frac{\tau_{stallSS}}{2}}{K_{TrmsSS}}\right)^2$$

$$P_{LT}\left(\frac{\omega_{NL1}}{2}\right) + P_R = 27.032\,W$$

$$P_T = 27.032\,W$$ Checks

Motor Constant

Using (48)

$$K_{MSS} := \sqrt{\frac{2}{3}}\cdot\frac{K_{TrmsSS}}{\sqrt{R_{\phi\phi}}}$$ $$K_{MSS} = 0.095\,\frac{N\cdot m}{\sqrt{W}}$$

Also

$$K_{MSS} := \frac{2}{\sqrt{3}}\cdot\frac{K_{TpSS}}{\sqrt{R_{\phi\phi}}}$$ $$K_{MSS} = \blacksquare\,\frac{N\cdot m}{\sqrt{W}}$$

Time constants

Using (52)

$$\tau_m := \frac{\sqrt{3}}{2}\cdot\frac{R_{\phi\phi}\cdot J_M}{K_{e\phi\phi p}\cdot K_{TpSS}}$$

Using (54)

$$\tau_e := \frac{L_{\phi\phi}}{R_{\phi\phi}}$$

Note No difference in either time constants published or caculated!

$$\tau_m = 1.494\,milli\cdot s$$ Checks

$$\tau_e = 1.429\,milli\cdot s$$ Checks

$$K_{TpSS}\cdot K_{e\phi\phi p} = 0.044\,\Omega\cdot\frac{N\cdot m}{\frac{rad}{s}}$$

From Attachment A, Chaart 1 Col 5.

Both Check

From Attachment A, Chaart 2 Col 10.

$$\frac{K_{TrmsSS}^2}{\sqrt{3}} = 0.044\,\Omega\cdot\frac{N\cdot m}{\frac{rad}{s}}$$

$$K_{TrmsSS}\cdot K_{e\phi\phi rmsSS} = 0.044\,\Omega\cdot\frac{N\cdot m}{\frac{rad}{s}}$$

Resistor power calculation using the Motor Constant, KM.

$$P_R = \left(\frac{\tau}{K_M}\right)^2$$ Derived from definition (102) "Page 15"

$$P_R := \left(\frac{\frac{\tau_{stallSS}}{2}}{K_M}\right)^2$$

$$P_R = 13.516\,W$$ Checks

Load power equation verifications

$$P_{LT} = \frac{\omega_{NL1}}{2}\cdot\frac{\tau_{stallSS}}{2} = 13.516\,W \qquad (\tau\cdot\omega)$$

From (25a)

$$P_{LTSS} := \frac{\sqrt{3}}{2}\cdot K_{e\phi\phi p}\cdot\left(\frac{\omega_{NL1}}{2}\right)\cdot\left(\frac{\frac{\tau_{stallSS}}{2}}{K_{TpSS}}\right)$$ with $I_{\phi\phi}p$ calculation $$I_{\phi\phi pSS} := \frac{\frac{\tau_{stallSS}}{2}}{K_{TpSS}}$$

$$P_{LTSS} = 13.516\,W$$ Check $$I_{\phi\phi pSS} = 1.794\,A\cdot p$$

From (25b)

$$P_{LTSS} := \sqrt{3}\cdot K_{e\phi\phi rmsSS}\cdot\left(\frac{\omega_{NL1}}{2}\right)\cdot\left(\frac{\frac{\tau_{stallSS}}{2}}{K_{TrmsSS}}\right)$$ with $I_{\phi\phi}rms$ calculation $$I_{\phi\phi rmsSS} := \frac{\frac{\tau_{stallSS}}{2}}{K_{TrmsSS}}$$

$$P_{LTSS} = 13.516\,W$$ Check $$I_{\phi\phi rmsSS} = 1.268\,A\cdot rms$$

$$\frac{I_{\phi\phi rmsSS}}{I_{\phi\phi pSS}} = \frac{1}{\sqrt{2}}$$ Verifies (A11)

DWG NO.

DIP30-07-001A

WINDING CONSTANTS *	UNITS	TOL	SYM	WDG A
DC RESISTANCE	OHMS	±12.5%	R	5.6
VOLTAGE @ T_P	VOLTS	NOMINAL	V_P	17.4
CURRENT @ T_P	AMPERES	NOMINAL	I_P	3.1
TORQUE SENSITIVITY	OZ-IN/AMP	±10%	K_T	32.0
BACK EMF CONSTANT	VOLTS/(RAD/SEC)	±10%	K_B	0.226
INDUCTANCE @ 1 KHz	MILLIHENRY	±30%	L	8.0

MOTOR PARAMETERS *	UNITS	SYM	NOM. VALUE
PEAK TORQUE	OZ-IN	T_P	100
MOTOR CONSTANT	OZ-IN/$\sqrt{\text{WATT}}$	K_M	13.5
ELECTRICAL TIME CONSTANT	MILLISECOND	τ_E	1.43
MECHANICAL TIME CONSTANT	MILLISECOND	τ_M	1.49
POWER I^2R @ T_P	WATTS	P_P	53.8
DAMPING FACTOR (ZERO IMPEDANCE)	OZ-IN/(RAD/SEC)	F_O	1.29
ROTOR INERTIA	OZ-IN SEC^2	J_M	1.923×10^{-3}
THEO. NO-LOAD SPEED @ 24 VDC	RAD/SEC	ω_O	106
THEO ACC @ T_P	RAD/SEC^2	α_T	5.2×10^4
THERMAL RESISTANCE	°C/WATT	θ_{TH}	5.2
MAX ALLOWABLE WINDING TEMP	°C	TEMP	155
NUMBER OF PHASES/WINDING TYPE			3/Y
NUMBER OF POLES			12
WEIGHT	OZ	W_T	7.5

* 25°C AMBIENT

DIRECTION OF ROTATION

CW VIEWED FROM LEAD SIDE