EXAMPLE 1 "Delta Winding" Sine / Sine and Sine / Square Evaluation.

Conversion factors:

$\text{ounce\_in} := 7.06155181 \cdot 10^{-3} \cdot \text{N} \cdot \text{m}$ $\quad 1\text{N m} = 141.6119\,\text{ounce\_in}$ $\quad \text{milli} := 10^{-3}$ $\quad \mu := 10^{-6}$

$\text{kg} \cdot \text{m}^2 = 141.6119\,\text{ounce\_in} \cdot \text{s}^2$ $\quad \text{krpm} := 104.72 \cdot \frac{\text{rad}}{\text{s}}$ $\quad \text{rev} := 2 \cdot \pi \cdot \text{rad}$

kp = kgf = kilopond force = kg force $\quad 1\,\frac{\text{rad}}{\text{s}} = 0.0095\,\text{krpm}$ $\quad \text{rpm} := \frac{\text{krpm}}{1000}$

$\text{kp} := 35.274 \cdot \frac{\text{oz}}{\text{kg}}$ $\quad \text{kgf} = 9.80665\,\text{N}$ $\quad \text{N} = 0.101972\,\text{kgf}$

$\frac{2.2046 \cdot \text{lb}}{1 \cdot \text{kg}} = 1$ $\quad \frac{35.2736 \cdot \text{oz}}{1\text{kg}} = 1$ $\quad \text{rms} := 1$ $\quad \text{p} := 1$ $\quad$ rms and peak abbreviations

$1 \cdot \frac{\text{N} \cdot \text{m}}{\text{A}} = 141.6119\,\frac{\text{ounce\_in}}{\text{A}}$ $\quad$ Kt units

$1 \cdot \frac{\text{V}}{\frac{\text{rad}}{\text{s}}} = 104.72\,\frac{\text{V}}{\text{krpm}}$ $\quad$ Ke units

$1 \cdot \frac{\text{ounce\_in}}{\text{krpm}} = 67.4327\,\mu \cdot \frac{\text{N} \cdot \text{m}}{\frac{\text{rad}}{\text{s}}}$ $\quad$ D units (mech friction torque)

Motor Example BEI DIP24-11-001A

Datasheet Parameters **[See Vendor Specifications on Page 9]**

The motor winding is Delta, and both sinusoidal and trapazoidal commutation parameters are listed.

Sinusoidal Specifications:

$R_{\phi\phi} := 34.9 \cdot \Omega$ $\quad L_{\phi\phi} := 38.3 \cdot \text{milli} \cdot \text{H}$ $\quad J_M := 0.00176 \cdot \text{ounce\_in} \cdot \text{s}^2$ $\quad P := 8$ $\quad \tau_e = 1.1 \cdot \text{milli} \cdot \text{s}$

$\tau_m = 2.52 \cdot \text{milli} \cdot \text{s}$

$K_{Tp} := 53.71 \cdot \frac{\text{ounce\_in}}{\text{A} \cdot \text{p}}$ $\quad K_{e\phi\phi p} := 0.438 \cdot \frac{\text{V} \cdot \text{p}}{\frac{\text{rad}}{\text{s}}}$

$K_{Trms} := 75.96 \cdot \frac{\text{ounce\_in}}{\text{A} \cdot \text{rms}}$ $\quad K_{e\phi\phi rms} := 0.310 \cdot \frac{\text{V} \cdot \text{rms}}{\frac{\text{rad}}{\text{s}}}$

$K_M := 10.5 \cdot \frac{\text{ounce\_in}}{\sqrt{\text{W}}}$ $\quad \omega_{NL\_18.7Vrms} = 60.32 \cdot \frac{\text{rad}}{\text{s}}$ $\quad$ **No Load Speed @ 18.7 Vrms**

$\omega_{L1} := 47.12 \cdot \frac{\text{rad}}{\text{s}}$ $\quad$ Speed @ 10 ounce_in and 18.6 Vrms.

$\omega_{L2} := 9 \cdot \frac{\text{rad}}{\text{s}}$ $\quad$ Speed @ 38 ounce_in and 18.0 Vrms.

Conversions to SI Units

$K_{Tp} = 0.3793 \frac{N \cdot m}{A \cdot p}$ (A is peak current)

$K_{Trms} = 0.5364 \frac{N \cdot m}{A \cdot rms}$ (A is rms current)

$K_{e\phi\phi p} := 0.438 \cdot \frac{V \cdot p}{\frac{rad}{s}}$ restated

$K_{e\phi\phi rms} := 0.310 \cdot \frac{V \cdot rms}{\frac{rad}{s}}$ restated

$K_M = 0.0741 \frac{N \cdot m}{\sqrt{W}}$

$J_M = 12.4283 \, \mu \cdot kg \cdot m^2$

Verification Sine / Sine row

$\frac{K_{Trms}}{K_{Tp}} = \sqrt{2}$ Attachment A, Chart 1 Col. 2

$\frac{K_{e\phi\phi rms}}{K_{e\phi\phi p}} = \frac{1}{\sqrt{2}}$ Attachment A, Chart 1 Col. 4

$\frac{K_{Tp}}{K_{e\phi\phi p}} = \frac{\sqrt{3}}{2}$ Attachment A, Chart 1 Col. 1

$\frac{K_{e\phi\phi rms}}{K_{Trms}} = \frac{1}{\sqrt{3}}$ Attachment A, Chart 1 Col. 3

Let

$K_{TpSS} := K_{Tp}$ $K_{TrmsSS} := K_{Trms}$ $K_{e\phi\phi rmsSS} := K_{e\phi\phi rms}$

Figures of Merit - Calculated

Stall Torque

$V_{\phi\phi rmstest} := 18.6 \cdot V \cdot rms$

$V_{\phi\phi ptest} := \sqrt{2} \, V_{\phi\phi rmstest}$ $V_{\phi\phi ptest} = 26.3044 \, V$

From (100) "Page 13" $\tau_{stall} := \frac{2}{\sqrt{3}} \frac{V_{\phi\phi ptest}}{R_{\phi\phi}} \cdot K_{Tp}$ (40) $I_{\phi\phi pstall} := \frac{2}{\sqrt{3}} \cdot \frac{V_{\phi\phi ptest}}{R_{\phi\phi}}$ $I_{\phi\phi pstall} = 0.8703 \, A \, p$

$I_{\phi\phi pstallSS} := I_{\phi\phi pstall}$

$\tau_{stall} = 0.3301 \, N \cdot m$ $\tau_{stall} = 46.7441 \, ounce\_in$

~ continuous stall torque
Spec'd at 46.1 ounce_in
with a 155 deg C winding
temperature

No load speed @ 18.7 Vp

$V_{\phi\phi rmstest2} := 18.7 \cdot V \cdot rms$ $V_{\phi\phi ptest2} := \sqrt{2} \cdot V_{\phi\phi rmstest2}$

Using (42) $\omega_{NL1} := \frac{V_{\phi\phi ptest2}}{K_{e\phi\phi p}}$ $\omega_{NL1} = 60.3785 \frac{rad}{s}$ Spec gives 60.32 rad/s Checks

Speed -Torque Curve

No load voltage test values:

All Check

$V_{\phi\phi rms1} := 18.7 \cdot V \cdot rms$ $\omega_{NL1} := \frac{V_{\phi\phi rms1}}{K_{e\phi\phi rms}}$ $\omega_{NL1} = 60.3226 \frac{rad}{s}$

$V_{\phi\phi rms2} := 18.6 \cdot V \cdot rms$ $\omega_{NL2} := \frac{V_{\phi\phi rms2}}{K_{e\phi\phi rms}}$ $\omega_{NL2} = 60 \frac{rad}{s}$

$V_{\phi\phi rms3} := 18.0 \cdot V \cdot rms$ $\omega_{NL3} := \frac{V_{\phi\phi rms3}}{K_{e\phi\phi rms}}$ $\omega_{NL3} = 58.0645 \frac{rad}{s}$

Per (42) $\frac{V_{\phi\phi p}}{K_{e\phi\phi p}} = \frac{V_{\phi\phi rms}}{K_{e\phi\phi rms}}$

Torque test values

$\tau_1 := 0 \cdot ounce\_in$ $\qquad \tau_1 = 0\,N \cdot m$

$\tau_2 := 10.0 \cdot ounce\_in$ $\qquad \tau_2 = 0.0706\,N \cdot m$

$\tau_3 := 38 \cdot ounce\_in$ $\qquad \tau_3 = 0.2683\,N \cdot m$

Speed - torque slope, Rm

Using (44) $\qquad R_{mSS} := \frac{\sqrt{3}}{2} \cdot \frac{R_{\phi\phi}}{K_{e\phi\phi p} \cdot K_{TpSS}}$ $\qquad R_{mSS} = 181.9394 \frac{\frac{rad}{s}}{N \cdot m}$

$\frac{\sqrt{3}}{2} \cdot \frac{R_{\phi\phi}}{K_{e\phi\phi rmsSS} \cdot K_{TrmsSS}} = 181.7646 \frac{\frac{rad}{s}}{N \cdot m}$ $\qquad$ Alternate form for Sine / Sine

Speed test points

Using (46) $\qquad$ All Check

$\omega_1 := \omega_{NL1} - R_{mSS} \cdot \tau_1$ $\qquad \omega_1 = 60.3226 \frac{rad}{s}$ $\qquad$ Spec is $\quad \omega_1 = 60.32 \cdot \frac{rad}{s}$

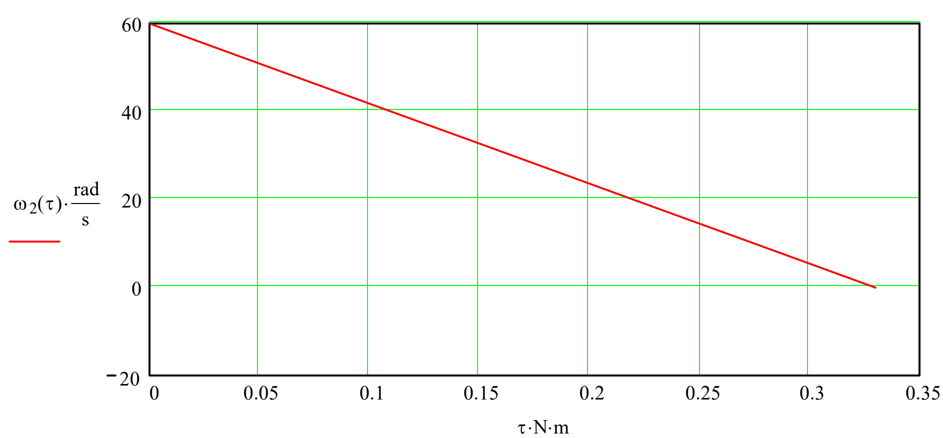
$\omega_2 := \omega_{NL2} - R_{mSS} \cdot \tau_2$ $\qquad \omega_2 = 47.1523 \frac{rad}{s}$ $\qquad$ Spec is $\quad \omega_1 = 47.12 \cdot \frac{rad}{s}$

$\omega_3 := \omega_{NL3} - R_{mSS} \cdot \tau_3$ $\qquad \omega_3 = 9.2431 \frac{rad}{s}$ $\qquad$ Spec is $\quad \omega_1 = 9.0 \cdot \frac{rad}{s}$

$\tau := 0 \cdot N \cdot m, (0.03 \cdot N \cdot m) .. 0.33 \cdot N \cdot m$ $\qquad$ Define range variable

Plot ω2 vs τ

Speed Torque Curve $\qquad \omega_2(\tau) := \omega_{NL2} - R_{mSS} \cdot \tau$ $\qquad$ Equation (46)

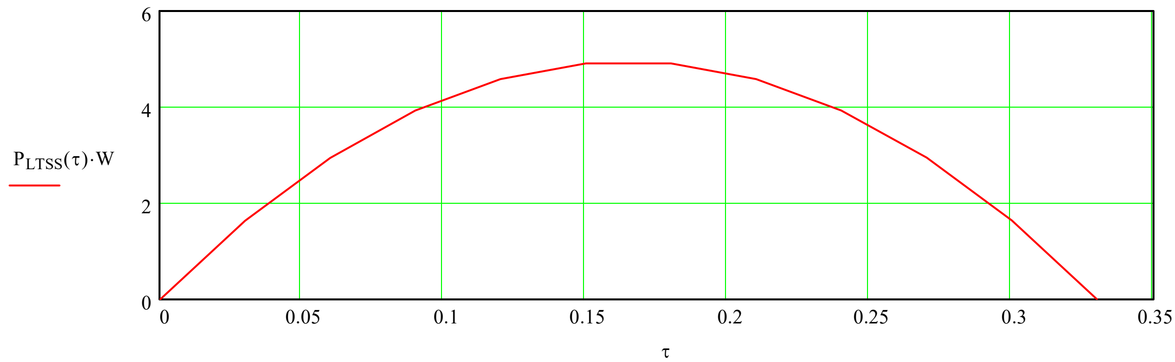


$\omega_2(0 \cdot N \cdot m) = 60 \frac{rad}{s}$ $\qquad \omega_2\left(\frac{\tau_{stall}}{2}\right) = 29.9722 \frac{rad}{s}$ $\qquad \omega_2(\tau_{stall}) = -0 \frac{rad}{s}$

Load Power as a function of torque:

$$P_{LTSS}(\tau) := -\left(\frac{\omega_{NL2}}{\tau_{stall}}\right)\cdot\tau^2 + \omega_{NL2}\cdot\tau$$

See Revisit (46) Page 10



$\omega^*\tau$

$$P_{LTSS}\left(\frac{\tau_{stall}}{2}\right) = 4.9513\ W \qquad \frac{\tau_{stall}}{2}\cdot\frac{\omega_{NL2}}{2} = 4.9513\ W$$

Checks

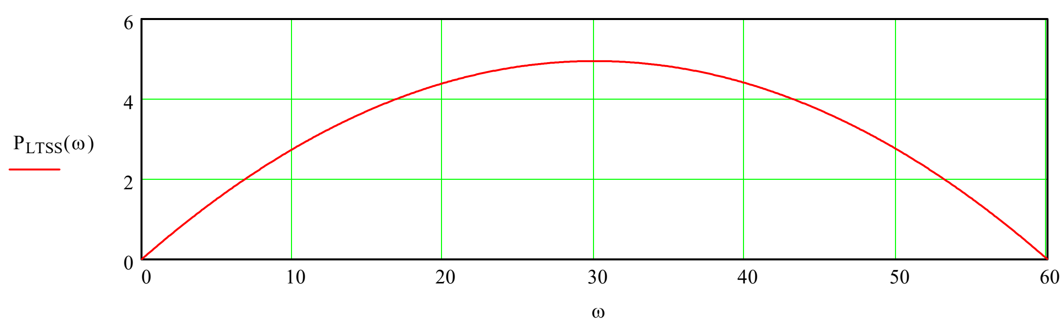
$$\omega := 0\cdot\frac{rad}{s}, \left(0.1\cdot\frac{rad}{s}\right) .. 60\cdot\frac{rad}{s}$$

Define range variable

Load Power as a function of speed:

$$P_{LTSS}(\omega) := -\left(\frac{\tau_{stall}}{\omega_{NL2}}\right)\cdot\omega^2 + \tau_{stall}\cdot\omega$$

See Revisit (46) Page 10



Using (27)

$$P_{RSS} := \frac{3}{2}\cdot\left(\frac{\frac{\tau_{stall}}{2}}{K_{TrmsSS}}\right)^2\cdot R_{\phi\phi}$$

with the argument solving for $I_{\phi\phi}$

$$\frac{\frac{\tau_{stall}}{2}}{K_{Trms}} = 0.3077\ A\cdot rms$$

$$P_{RSS} = 4.9561\ W$$

Using (14) + (15)

$$P_{LTSS} := 4.9513\cdot W \qquad P_{RSS} = 4.9561\ W$$

$$P_T := \frac{\omega_{NL2}}{2}\cdot\frac{\tau_{stall}}{2} + \frac{3}{2}\cdot R_{\phi\phi}\cdot\left(\frac{\frac{\tau_{stall}}{2}}{K_{TrmsSS}}\right)^2$$

$$P_T = 9.9074\ W \qquad P_{LTSS} + P_{RSS} = 9.9074\ W$$

Checks

Using (48)

$$K_{MSS} := \sqrt{\frac{2}{3}} \cdot \frac{K_{Trms}}{\sqrt{R_{\phi\phi}}} \qquad K_{MSS} = 0.0741 \frac{N \cdot m}{\sqrt{W}}$$ Checks $$\frac{2}{\sqrt{3}} \cdot \frac{K_{Tp}}{\sqrt{R_{\phi\phi}}} = 0.0741 \frac{N \cdot m}{\sqrt{W}}$$ Checks

Using (52)

$$\tau_{mSS} := \frac{\sqrt{3}}{2} \frac{R_{\phi\phi} \cdot J_M}{K_{e\phi\phi p} \cdot K_{TpSS}}$$

Using (54)

$$\tau_e := \frac{L_{\phi\phi}}{R_{\phi\phi}}$$

$$\tau_{mSS} = 2.2612 \,\text{milli} \cdot s \qquad \tau_{mSS} := \frac{\sqrt{3}}{2} \cdot \frac{R_{\phi\phi} \cdot J_M}{K_{e\phi\phi rms} \cdot K_{Trms}}$$ Using relationship in Attachment A, Chart 2, Col. 9

$$\tau_{mSS} = 2.259 \,\text{milli} \cdot s \qquad \tau_e = 1.0974 \,\text{milli} \cdot s$$ Checks

** τ_m **=** $2.52 \text{milli} \cdot s$ **Different** Published; typo?.

Find $$K_{TpSS} \cdot K_{e\phi\phi p} = 0.1661 \, \Omega \cdot \frac{N \cdot m}{\frac{rad}{s}}$$

From Attachment A Chart 1 Col. 5

$$\frac{K_{TrmsSS}^{\,2}}{\sqrt{3}} = 0.1661 \, \Omega \cdot \frac{N \cdot m}{\frac{rad}{s}}$$

From Attachment A Chart 2 Col 10

$$K_{TrmsSS} \cdot K_{e\phi\phi rmsSS} = 0.1663 \, \Omega \cdot \frac{N \cdot m}{\frac{rad}{s}}$$

Both Check.

Resistor power calculation using the Motor Constant, KM,

$$P_R = \left(\frac{\tau}{K_M} \right)^2$$ Derived from definition (102) "Page 15"

$$P_{RSS} := \left(\frac{\frac{\tau_{stall}}{2}}{K_M} \right)^2$$

$$P_{RSS} = 4.9547 \, W$$ Checks

Load power equation verifications. $P_{LTSS} = \frac{\omega_{NL2}}{2} \cdot \left(\frac{\tau_{stall}}{2}\right) = 4.9513\,W$ $(\tau \cdot \omega)$

From (25a)

$$P_{LTSS} := \frac{\sqrt{3}}{2} \cdot K_{e\phi\phi p} \cdot \left(\frac{\omega_{NL2}}{2}\right) \cdot \left(\frac{\frac{\tau_{stall}}{2}}{K_{TpSS}}\right)$$

with $I\phi\phi p$ calculation $I_{\phi\phi pSS} := \frac{\frac{\tau_{stall}}{2}}{K_{TpSS}}$

$P_{LTSS} = 4.9519\,W$ Acceptable Check $I_{\phi\phi pSS} = 0.4352\,A \cdot p$

From (25b)

$$P_{LTSS} := \sqrt{3} \cdot K_{e\phi\phi rmsSS} \cdot \left(\frac{\omega_{NL2}}{2}\right) \cdot \left(\frac{\frac{\tau_{stall}}{2}}{K_{TrmsSS}}\right)$$

with $I\phi\phi rms$ calculation $I_{\phi\phi rmsSS} := \frac{\frac{\tau_{stall}}{2}}{K_{TrmsSS}}$

$P_{LTSS} = 4.9563\,W$ Acceptable Check $I_{\phi\phi rmsSS} = 0.3077\,A \cdot rms$

$$\frac{I_{\phi\phi rmsSS}}{I_{\phi\phi pSS}} = \frac{1}{\sqrt{2}}$$ Verifies (A11)

Squarewave Calculations:

To this point all parameters and figures of merit are dead on except for the mechanical time constant.

$K_{e\phi\phi p} := 0.438 \cdot \frac{V \cdot p}{\frac{rad}{s}}$ $K_{e\phi\phi p} = 62.026 \frac{ounce\_in}{A \cdot p}$ $=$ K_{Tp} **BEI assumes KTp=Keϕϕp which is not true!**

From (A16)

$K_{TpSQ} := \frac{3}{\pi} \cdot K_{e\phi\phi p}$ $K_{TpSQ} = 0.4183 \frac{N \cdot m}{A \cdot p}$ **Verifying Attachment A, Chart 1, Col. 1. for Sine / Square row.**

From (53) $\tau_{mSQ} := \frac{R_{\phi\phi} \cdot J_M}{K_{TpSQ} \cdot K_{e\phi\phi p}}$ $\tau_{mSQ} = 2.3677\,milli \cdot s$

This is slightly slower than a pure sinewave drive.

$$\frac{\tau_{mSS}}{\tau_{mSQ}} = \frac{2.259 \cdot milli \cdot s}{2.37 \cdot milli \cdot s} \sim \frac{3}{\pi} = 0.9549$$

Sine / Square Parameters

Verification Sine / Square row

$$K_{e\phi\phi rmsSQ} := \frac{K_{e\phi\phi p}}{\sqrt{2}}$$ $$K_{e\phi\phi rmsSQ} = 0.3097\,\frac{V\cdot rms}{\frac{rad}{s}}$$ Attachment A, Chart 1 Col. 4

(A18) $$K_{TrmsSQ} := \frac{3}{\pi}\cdot\sqrt{\frac{3}{2}}\cdot K_{e\phi\phi p}$$ $$K_{TrmsSQ} = 0.5123\,\frac{N\cdot m}{A\cdot rms}$$ Attachment A, Chart 1 Col.1 X Col. 2

$$K_{TrmsSQ} := \sqrt{\frac{3}{2}}\,K_{TpSQ}$$ $$K_{TrmsSQ} = 0.5123\,\frac{N\cdot m}{A\cdot rms}$$ Attachment A, Chart 1 Col. 2

$$K_{TrmsSQ} := \frac{3}{\pi}\cdot\sqrt{3}\cdot K_{e\phi\phi rms}$$ $$K_{TrmsSQ} = 0.5127\,\frac{N\cdot m}{A\cdot rms}$$ Attachment A, Chart 1 INV (Col.3)

$$K_{TpSQ} = 0.4183\,\frac{N\cdot m}{A\cdot p}$$

$$K_{TrmsSQ} = 72.6095\,\frac{ounce\_in}{A\cdot rms}$$ Which is expected $$\frac{K_{TpSQ}\cdot K_{e\phi\phi p}}{\frac{\pi}{3}\cdot\frac{2}{3}\cdot K_{TrmsSQ}^{2}} = 0.9981$$ Attachment A, Chart 1 Col. 5

Since

$$K_{TrmSS} := 0.5364\cdot\frac{N\cdot m}{A\cdot rms}$$

$$K_{TrmsSQ} = 0.5127\,\frac{N\cdot m}{A\cdot rms}$$

$$\frac{\frac{\sqrt{3}}{2}K_{TpSQ}\cdot K_{e\phi\phi p}}{K_{TrmsSQ}\cdot K_{e\phi\phi rmsSQ}} = 0.9991$$ Attachment A, Chart 2 Col 10

$$K_{TrmsSS} = 75.96\,\frac{ounce\_in}{A\cdot rms}$$

$$\frac{K_{TrmsSQ}}{K_{TrmsSS}} = 0.9559 \quad \sim \quad 3/\pi$$ Verify (A27)

$$\frac{K_{TpSQ}}{K_{TpSS}} = 1.1028 \quad \sim \quad \frac{2\cdot\sqrt{3}}{\pi}$$ Verify (A26)

Figures of Merit - Calculated

From Spec Sheet $$V_{\phi\phi ptestSQ} := 22.5\cdot V\cdot p$$

Stall Torque

Using (101)

"Page 14" $$\tau_{stallSQ} := \frac{V_{\phi\phi ptestSQ}}{R_{\phi\phi}}\cdot K_{TpSQ}$$

$$\tau_{stallSQ} = 0.2697\,N\cdot m$$ $$\tau_{stallSQ} = 38.1858\,ounce\_in$$ Using (41) $$I_{\phi\phi pstallSQ} := \frac{\tau_{stallSQ}}{K_{TpSQ}}$$

$$I_{\phi\phi pstallSQ} = 0.6447\,A\cdot p$$

Spec is 0.65 Ap Checks

Note that the spec is written as Current at τp; Voltage @ τp.

I_p Vp

No Load Speed

From (42) $$\omega_{NLSQ} := \frac{V_{\phi\phi ptestSQ}}{K_{e\phi\phi p}}$$ $$\omega_{NLSQ} = 51.3699\,\frac{rad}{s}$$

Using (49)

$$K_{MSQ} := \frac{K_{TpSQ}}{\sqrt{R_{\phi\phi}}}$$

$K_{MSQ} = 0.0708 \frac{N \cdot m}{\sqrt{W}}$ $\quad K_{MSQ} = 10.0261 \frac{ounce\_in}{\sqrt{W}}$ Checks $\frac{3}{\pi}$ reduction

ALSO $\sqrt{\frac{2}{3}} \cdot \frac{K_{TrmsSQ}}{\sqrt{R_{\phi\phi}}} = 0.0709 \frac{N \cdot m}{\sqrt{W}}$ Checks Attachment A, Chart 2 Col 7 Sine / Square row

Note we want to use the SS values for Stall torque / 2 and the No Load Speed / 2 so a direct compaison can be made. As RmSS < RmSQ we adjust Vϕϕp so the two curves intersect at this point.

(102) "Page 15"

$$P_{RSQ} := \left(\frac{\frac{\tau_{stall}}{2}}{K_{MSQ}} \right)^2 \quad P_{RSQ} = 5.4341 \text{ W}$$

Using (34)

$$P_{RSQ} := \left(\frac{\frac{\tau_{stall}}{2}}{K_{TpSQ}} \right)^2 \cdot R_{\phi\phi} \quad P_{RSQ} = 5.4341 \text{ W}$$

Match!

Note the ratio of resistor power loss from a sine drive to a squarewave drive.

$P_{RSS} = 4.9547 \text{ W}$ For the same torque. = 0.9112823

$P_{RSQ} = 5.4341 \text{ W}$

$\frac{P_{RSS}}{P_{RSQ}} = 0.9118$ $\quad \sqrt{\frac{P_{RSS}}{P_{RSQ}}}$ = 0.9546 ~ 3 / π **To be expected!**

Load power equation verifications.

Using (45) $\quad R_{mSQ} := \frac{R_{\phi\phi}}{K_{e\phi\phi p} \cdot K_{TpSQ}} \quad R_{mSQ} = 190.5048 \frac{\frac{rad}{s}}{N \cdot m}$

From (46)

$\omega := \frac{\omega_{NL2}}{2} \quad \omega = \omega_{NLx} - R_{mSQ} \cdot \tau$

$\omega_{NLx} := \omega + R_{mSQ} \cdot \frac{\tau_{stall}}{2} \quad V_{\phi\phi ptestSQ} := \omega_{NLx} \cdot K_{e\phi\phi p}$

So

$\omega_{NLx} = 61.4415 \frac{rad}{s} \quad \tau := \frac{\tau_{stall}}{2} \quad V_{\phi\phi ptestSQ} = 26.9114 \text{ V} \cdot \text{p}$ **To match SS speed-torque point.**

From (32a)

$P_{LTSQ} = (\omega) \cdot \tau = 4.9513 \text{ W}$

$P_{LTSQ} := \left(\frac{3}{\pi} \right) \cdot K_{e\phi\phi p} \cdot \omega \cdot \frac{\tau}{K_{TpSQ}}$ with Iϕϕp calculation $\quad I_{\phi\phi pSQ} := \frac{\tau}{K_{TpSQ}}$

$P_{LTSQ} = 4.9513 \text{ W}$ Check $\quad I_{\phi\phi pSQ} = 0.3946 \text{ A} \cdot \text{p}$

From (32b)

$P_{LTSQ} := \frac{3}{\pi} \sqrt{3} \cdot K_{e\phi\phi rms} \cdot \omega \cdot \frac{\tau}{K_{TrmsSQ}}$ with Iϕϕrms calculation $\quad I_{\phi\phi rmsSQ} := \frac{\tau}{K_{TrmsSQ}}$

$P_{LTSQ} = 4.9513 \text{ W}$ Check $\quad I_{\phi\phi rmsSQ} = 0.3219 \text{ A} \cdot \text{rms}$

$\frac{I_{\phi\phi rmsSQ}}{I_{\phi\phi pSQ}} = 0.8157 \quad = \quad \sqrt{\frac{2}{3}}$ Verifies (A17)

BEI DIP24-11-001A

WINDING CONSTANTS *	UNITS	TOL	SYM	WDG A
VOLTAGE @ T_P	VOLTS	NOMINAL	V_P	22.5
CURRENT @ T_P	AMPERES	NOMINAL	I_P	.65
TORQUE SENSITIVITY	OZ–IN/AMP	±10%	K_T	62.0
* 25°C AMBIENT & TRAPEZOIDAL COMMUTATION				

WINDING CONSTANTS *	UNITS	TOL	SYM	WDG A
DC RESISTANCE	OHMS	±12.5%	R	34.9
CURRENT @ T_P	AMPERES	NOMINAL	I_P	.74
TORQUE SENSITIVITY (PK)	OZ–IN/AMP	±10%	K_T	53.71
TORQUE SENSITIVITY (RMS)	OZ–IN/AMP	±10%		75.96
BACK EMF CONSTANT (PK)	VOLTS/(RAD/SEC)	±10%	K_B	.438
BACK EMF CONSTANT (RMS)	VOLTS/(RAD/SEC)	±10%		.310
INDUCTANCE	MILLIHENRY	±30%	L	38.3
* 25°C AMBIENT & SINUSOIDAL COMMUTATION				

MOTOR PARAMETERS *	UNITS	SYM	NOM. VALUE
PEAK TORQUE	OZ–IN	T_P	120
CONTINUOUS STALL TORQUE **	OZ–IN	T_{CS}	46.1
MOTOR CONSTANT	OZ–IN/$\sqrt{\text{WATT}}$	K_M	10.5
ELECTRICAL TIME CONSTANT	MILLISECOND	τ_E	1.10
MECHANICAL TIME CONSTANT	MILLISECOND	τ_M	2.52
POWER .75 I^2R @ T_P	WATTS	P_P	14.6
DAMPING FACTOR (ZERO IMPEDANCE)	OZ–IN/(RAD/SEC)	F_O	.67
BREAKAWAY TORQUE	OZ–IN	T_F	.55 MAX
ROTOR INERTIA	OZ–IN SEC^2	J_M	0.00176
THEO. NO–LOAD SPEED @ 18.7 V(RMS)	RAD/SEC	ωNL_T	60.32
SPEED @ 10 OZ–IN & 18.6 V(RMS)	RAD/SEC	ω_{L1}	47.12
SPEED @ 38 OZ–IN & 18.0 V(RMS)	RAD/SEC	ω_{L2}	9.0
THEO ACC @ T_P	RAD/SEC^2	α_T	22727
THERMAL RESISTANCE (HOUSED)	°C/WATT	θ_{TH}	4.5
MAX ALLOWABLE WINDING TEMP	°C	TEMP	155
NUMBER OF PHASES/WINDING TYPE			3/Δ
NUMBER OF POLES			8
WEIGHT	OZ	W_T	9.3

* 25°C AMBIENT & SINUSOIDAL COMMUTATION
** 25°C AMBIENT AND 155°C WINDING TEMPERATURE

Revist (46)

$$\omega(\tau) = \omega_{NL} - R_m \cdot \tau$$

$$R_m = \frac{\omega_{NL}}{\tau_{stall}}$$

Let

$$\omega = (\tau_{stall} - \tau) \cdot \frac{\omega_{NL}}{\tau_{stall}}$$

$$\tau = \tau_{stall} - \frac{\omega \cdot \tau_{stall.}}{\omega_{NL}}$$

Since $P_{LT} = \tau \cdot \omega$

$$P_{LT}(\tau) = -\left(\frac{\omega_{NL}}{\tau_{stall}}\right) \cdot \tau^2 + \omega_{NL} \cdot \tau$$ Load power as a function of load torque

$$P_{LT}(\omega) = -\left(\frac{\tau_{stall}}{\omega_{NL}}\right) \cdot \omega^2 + \tau_{stall} \cdot \omega$$ Load power as a function of speed

MAX Power when

$\tau = \frac{1}{2} \cdot \tau_{stall}$ and $\omega = \frac{1}{2} \cdot \omega_{NL}$

How to find ---

Take derivatives of above and set equal to zero and solve for τ and ω which yield the maximum power.

$$\frac{d}{d\tau}\left[-\left(\frac{\omega_{NL}}{\tau_{stall}}\right) \cdot \tau^2 + \omega_{NL} \cdot \tau\right] = -2 \cdot \frac{\omega_{NL}}{\tau_{stall}} \cdot \tau + \omega_{NL} = 0$$

Solving for τ

$$\tau = \frac{\tau_{stall}}{2}$$

$$\frac{d}{d\omega}\left[-\left(\frac{\tau_{stall}}{\omega_{NL}}\right) \cdot \omega^2 + \tau_{stall} \cdot \omega\right] = -2 \cdot \frac{\tau_{stall}}{\omega_{NL}} \cdot \omega + \tau_{stall} = 0$$

Solving for ω

$$\omega = \frac{\omega_{NL}}{2}$$