

ATTACHMENT A

Torque Calculations, Parameter Comparisons, and Chart Summaries 1 and 2

Torque Calculations and Voltage and Current Conversions

Throughout

Torque = τ

$$K_{e\phi\phi p} = \frac{e_{B\phi\phi p}}{\omega}$$

$$K_{e\phi\phi rms} = \frac{e_{B\phi\phi rms}}{\omega}$$

$$K_{Tp} = \frac{\tau}{I_{\phi\phi p}}$$

$$K_{Trms} = \frac{\tau}{I_{\phi\phi rms}}$$

MOTOR / DRIVE

Square / Square [Refer to (29)]

$$\tau_{QQ} = K_{e\phi\phi p} \cdot I_{\phi\phi p} \quad (A1)$$

$$e_{B\phi\phi rms} = \frac{\sqrt{5}}{3} \cdot e_{B\phi\phi p} \quad (A3)$$

$$I_{\phi\phi rms} = \sqrt{\frac{2}{3}} \cdot I_{\phi\phi p} \quad (A5)$$

$$\tau_{QQ} = \sqrt{3} \cdot \sqrt{\frac{9}{10}} \cdot K_{e\phi\phi rms} \cdot I_{\phi\phi rms} \quad (A2)$$

$$K_{Tp} = K_{e\phi\phi p} \quad (A4)$$

$$K_{Trms} = \sqrt{\frac{3}{2}} \cdot K_{\phi\phi p} \quad (A6)$$

Sine / Sine [Refer to (26)]

$$\tau_{SS} = \frac{\sqrt{3}}{2} \cdot K_{e\phi\phi p} \cdot I_{\phi\phi p} \quad (A7)$$

$$e_{B\phi\phi rms} = \frac{e_{B\phi\phi p}}{\sqrt{2}} \quad (A9)$$

$$I_{\phi\phi rms} = \frac{I_{\phi\phi p}}{\sqrt{2}} \quad (A11)$$

$$\tau_{SS} = \sqrt{3} \cdot K_{e\phi\phi rms} \cdot I_{\phi\phi rms} \quad (A8)$$

$$K_{Tp} = \frac{\sqrt{3}}{2} \cdot K_{e\phi\phi p} \quad (A10)$$

$$K_{Trms} = \sqrt{\frac{3}{2}} \cdot K_{\phi\phi p} \quad (A12)$$

Sine / Square [Refer to (33)]

$$\tau_{SQ} = \frac{3}{\pi} \cdot K_{e\phi\phi p} \cdot I_{\phi\phi p} \quad (A13)$$

$$e_{B\phi\phi rms} = \frac{e_{B\phi\phi p}}{\sqrt{2}} \quad (A15)$$

$$I_{\phi\phi rms} = \sqrt{\frac{2}{3}} \cdot I_{\phi\phi p} \quad (A17)$$

$$\tau_{SQ} = \frac{3}{\pi} \cdot \sqrt{3} \cdot K_{e\phi\phi rms} \cdot I_{\phi\phi rms} \quad (A14)$$

$$K_{Tp} = \frac{3}{\pi} \cdot K_{e\phi\phi p} \quad (A16)$$

$$K_{Trms} = \frac{3}{\pi} \cdot \sqrt{\frac{3}{2}} \cdot K_{\phi\phi p} \quad (A18)$$

Square / Sine [Refer to (37)]

$$\tau_{QS} = \left(\frac{3}{\pi}\right)^2 \cdot K_{e\phi\phi p} \cdot I_{\phi\phi p} \quad (A19)$$

$$e_{B\phi\phi rms} = \left(\frac{3}{\pi}\right)^2 \cdot \sqrt{\frac{2}{3}} \cdot e_{B\phi\phi p} \quad (A21)$$

$$I_{\phi\phi rms} = \frac{I_{\phi\phi p}}{\sqrt{2}} \quad (A23)$$

$$\tau_{QS} = \sqrt{3} \cdot K_{e\phi\phi rms} \cdot I_{\phi\phi rms} \quad (A20)$$

$$K_{Tp} = \left(\frac{3}{\pi}\right)^2 \cdot K_{e\phi\phi p} \quad (A22)$$

$$K_{Trms} = \left(\frac{3}{\pi}\right)^2 \cdot \sqrt{2} \cdot K_{\phi\phi p} \quad (A24)$$

Comparison of Motor / Drive = Sine / Sine with Sine / Square

Relationships

When RMS currents are equal --

$$\frac{I_{\phi\phi pSS}}{\sqrt{2}} = \sqrt{\frac{2}{3}} \cdot I_{\phi\phi pSQ} \quad [(A11) = (A17)]$$

Then

$$I_{\phi\phi pSS} = \frac{2}{\sqrt{3}} \cdot I_{\phi\phi pSQ} \quad (A25) \quad \text{Relating peak currents.}$$

Resistor power loss --

$$P_{RSS} = \frac{3}{2} \cdot I_{\phi\phi rmsSS}^2 \cdot R_{\phi\phi} = \frac{3}{2} \frac{I_{\phi\phi pSS}^2}{2} \cdot R_{\phi\phi} \quad [\text{Refer to (27)}]$$

$$P_{RSQ} = I_{\phi\phi pSQ}^2 \cdot R_{\phi\phi} \quad [\text{Refer to (34)}]$$

Equating resistor power loss --

$$I_{\phi\phi pSS} = I_{\phi\phi pSQ} \cdot \sqrt{\frac{4}{3}} = I_{\phi\phi pSQ} \cdot \frac{2}{\sqrt{3}}$$

$$\text{Let} \quad K_{e\phi\phi pSS} = K_{e\phi\phi pSQ} = K_{e\phi\phi p}$$

$$\frac{K_{TpSS}}{K_{e\phi\phi p}} = \frac{\sqrt{3}}{2} \quad \text{From (A10)}$$

$$\frac{K_{TpSQ}}{K_{e\phi\phi p}} = \frac{3}{\pi} \quad \text{From (A16)}$$

$$\frac{K_{TpSQ}}{K_{TpSS}} = \frac{2 \cdot \sqrt{3}}{\pi} \quad (A26)$$

$$\frac{2 \cdot \sqrt{3}}{\pi} = 1.1027$$

$$K_{TrmsSS} = \sqrt{\frac{3}{2}} \cdot K_{e\phi\phi p} \quad \text{From (A12)}$$

$$K_{TrmsSQ} = \frac{3}{\pi} \cdot \sqrt{\frac{3}{2}} \cdot K_{e\phi\phi p} \quad \text{From (A18)}$$

$$\frac{K_{TrmsSQ}}{K_{TrmsSS}} = \frac{3}{\pi} \quad (A27)$$

$$\frac{3}{\pi} = 0.95493$$

Torque generated is less when a sinewave motor is driven with a squarewave drive instead of a sinewave drive; the rms currents being equal.

Comparison of Motor / Drive = Square / Square with Square / Sine

When rms currents are equal - -

$$\frac{I_{\phi\phi pQS}}{\sqrt{2}} = \sqrt{\frac{2}{3}} \cdot I_{\phi\phi pQQ} \quad [(A23) = (A5)]$$

Then

$$I_{\phi\phi pQS} = \frac{2}{\sqrt{3}} \cdot I_{\phi\phi pQQ} \quad (A28)$$

Relating peak currents.

$$\frac{K_{TpQQ}}{K_{e\phi\phi p}} = 1 \quad \text{From (A4)}$$

$$\frac{K_{TpQS}}{K_{e\phi\phi p}} = \left(\frac{3}{\pi} \right)^2 \quad \text{From (A22)}$$

$$\frac{K_{TpQS}}{K_{TpQQ}} = \left(\frac{3}{\pi} \right)^2 \quad (A29)$$

$$\left(\frac{3}{\pi} \right)^2 = 0.9119$$

$$K_{TrmsQQ} = \sqrt{\frac{3}{2}} \cdot K_{e\phi\phi p} \quad \text{From (A6)}$$

$$K_{TrmsQS} = \left(\frac{3}{\pi} \right)^2 \cdot \sqrt{2} \cdot K_{e\phi\phi p} \quad \text{From (A24)}$$

$$\frac{K_{TrmsQS}}{K_{TrmsQQ}} = \frac{6}{\pi^2} \cdot \sqrt{3} \quad (A30)$$

$$\frac{6}{\pi^2} \cdot \sqrt{3} = 1.05296$$

Torque generated is increased when a squarewave motor is driven with a sinewave drive instead of a squarewave drive; the rms currents being equal.

CHART 1

PARAMETERS										
Motor / Drive	Column 1		Column 2		Column 3		Column 4		Column 5	
	$\frac{K_{Tp}}{K_{\phi\phi p}}$		$\frac{K_{Tms}}{K_{Tp}}$		$\frac{K_{\phi\phi rms}}{K_{Tms}}$		$\frac{K_{\phi\phi rms}}{K_{\phi\phi p}}$		$K_{Tp} \cdot K_{\phi\phi p}$	
Square / Square	1		$\sqrt{\frac{3}{2}}$		$\sqrt{\frac{10}{9}} \cdot \frac{1}{\sqrt{3}}$		$\frac{\sqrt{5}}{3}$		$\frac{2}{3} \cdot K_{Tms}^2$	
Sine / Sine	$\frac{\sqrt{3}}{2}$		$\sqrt{2}$		$\frac{1}{\sqrt{3}}$		$\frac{1}{\sqrt{2}}$		$\frac{K_{Tms}^2}{\sqrt{3}}$	
Sine / Square	$\frac{3}{\pi}$		$\sqrt{\frac{3}{2}}$		$\frac{\pi}{3} \cdot \frac{1}{\sqrt{3}}$		$\frac{1}{\sqrt{2}}$		$\frac{\pi}{3} \cdot \frac{2}{3} K_{Tms}^2$	
Square / Sine	$\left(\frac{3}{\pi}\right)^2$		$\sqrt{2}$		$\frac{1}{\sqrt{3}}$		$\left(\frac{3}{\pi}\right)^2 \cdot \sqrt{\frac{2}{3}}$		$\left(\frac{\pi}{3}\right)^2 \cdot \frac{K_{Tms}^2}{2}$	

CHART 2			FIGURES OF MERIT					
Motor / Drive			Column 6	Column 7	Column 8	Column 9	Column 10	
			P_R	K_M	R_m	τ_m	$\frac{K_{T_{rms}} \cdot K_{\phi\phi_{rms}}}{K_{T_P} \cdot K_{\phi\phi_P}}$	
Square / Square			* $R_{\phi\phi} \cdot \left(\frac{\tau}{K_{TP}} \right)^2$	** $\frac{K_{TP}}{\sqrt{R_{\phi\phi}}}$	$\frac{R_{\phi\phi}}{K_{TP} \cdot K_{\phi\phi_P}}$	$\frac{R_{\phi\phi} \cdot J}{K_{TP} \cdot K_{\phi\phi_P}}$	$\sqrt{\frac{5}{6}}$	
Sine / Sine			$\frac{3}{2} R_{\phi\phi} \cdot \left(\frac{\tau}{K_{T_{rms}}} \right)^2$	$\sqrt{\frac{2}{3}} \cdot \frac{K_{T_{rms}}}{\sqrt{R_{\phi\phi}}}$	$\frac{\sqrt{3}}{2} \cdot \frac{R_{\phi\phi}}{K_{TP} \cdot K_{\phi\phi_P}}$	$\frac{\sqrt{3}}{2} \cdot \frac{R_{\phi\phi} \cdot J}{K_{TP} \cdot K_{\phi\phi_P}}$	1	
Sine / Square			* $R_{\phi\phi} \cdot \left(\frac{\tau}{K_{TP}} \right)^2$	** $\frac{K_{TP}}{\sqrt{R_{\phi\phi}}}$	$\frac{R_{\phi\phi}}{K_{TP} \cdot K_{\phi\phi_P}}$	$\frac{R_{\phi\phi} \cdot J}{K_{TP} \cdot K_{\phi\phi_P}}$	$\frac{\sqrt{3}}{2}$	
Square / Sine			$\frac{3}{2} R_{\phi\phi} \cdot \left(\frac{\tau}{K_{T_{rms}}} \right)^2$	$\sqrt{\frac{2}{3}} \cdot \frac{K_{T_{rms}}}{\sqrt{R_{\phi\phi}}}$	$\frac{\sqrt{3}}{2} \cdot \frac{R_{\phi\phi}}{K_{TP} \cdot K_{\phi\phi_P}}$	$\frac{\sqrt{3}}{2} \cdot \frac{R_{\phi\phi} \cdot J}{K_{TP} \cdot K_{\phi\phi_P}}$	$\left(\frac{3}{\pi} \right)^2 \cdot \frac{2}{\sqrt{3}}$	
ALSO			* $= \frac{3}{2} \cdot R_{\phi\phi} \cdot \left(\frac{\tau}{K_{T_{rms}}} \right)^2$	** $= \sqrt{\frac{2}{3}} \cdot \frac{K_{T_{rms}}}{\sqrt{R_{\phi\phi}}}$				